Accelerators

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Colliders or Fixed Targets?

If you want to do a scattering experiment of a moving particle **1** on particle **2** you have two options:

1. Particle **2** is at rest (beam on a target)
2. Particle **2** is moving in the opposite direction of particle **1**. Particles **1** and **2** collide 'head on'.

Which is the most effective option?

Reminder: $m_1^2 = E_1^2 - \boldsymbol{p_1^2}$ .Total energy squared $s = \ E_{CM}^2 = (E_1 + E_2)^2 - (\boldsymbol{p_1} + \boldsymbol{p_2})^2$

Particle 1	Particle 2		s
$p_1=(E_1,\mathbf{p}_1)$	At rest	$p_2=(m_2,\mathbf{0})$	$(E_1 + m_2)^2 - (\boldsymbol{p_1} + \boldsymbol{0})^2 = \left(E_1^2 + m_2^2 + 2E_1 \cdot m_2 - \boldsymbol{p_1^2}\right) = m_1^2 + m_2^2 + 2E_1 \cdot m_2$
$p_1=(E_1,\mathbf{p}_1)$	In motion	$p_2=(E_2,\mathbf{p}_2)$	$(E_1 + E_2)^2 - (\boldsymbol{p_1} + \boldsymbol{p_2})^2 = E_1^2 + E_2^2 + 2E_1 \cdot E_2 - \boldsymbol{p_1^2} - \boldsymbol{p_2^2} - \boldsymbol{2 \cdot p_1 \cdot p_2}$ $= m_1^2 + m_2^2 + 2E_1 \cdot m_2 - 2 \cdot \boldsymbol{p_1} \cdot \boldsymbol{p_2}$
$p_1=(E,\mathbf{p})$	In motion	$p=(E,-\mathbf{p})$	$4E^2$

In practice, neglecting masses, a beam on a target $E_{CM}^{target} = \sqrt{s} = \sqrt{2E_{beam} \cdot m_2}$ *while* $E_{CM}^{Collider} = \sqrt{s} = 2E_{beam}$

→ ECM goes with the $\sqrt{(2E_{beam}m)}$ for beams on a target and like $2E_{beam}$ for Colliders

Accelerators (→ Colliders)

A few starting comments:

Accelerators with equal energy beams are most effective. ***Linear vs Circular***

- ***Linear Colliders*** may be an option for at least some particle types; however accelerated particles meet each other only once and then are lost. But they represent ~a good option if *very high energy electron beams* are needed (see later).
- ***Circular Colliders*** (they are closed Colliders) give the important possibility of colliding beams a huge number of times.

Limitations:

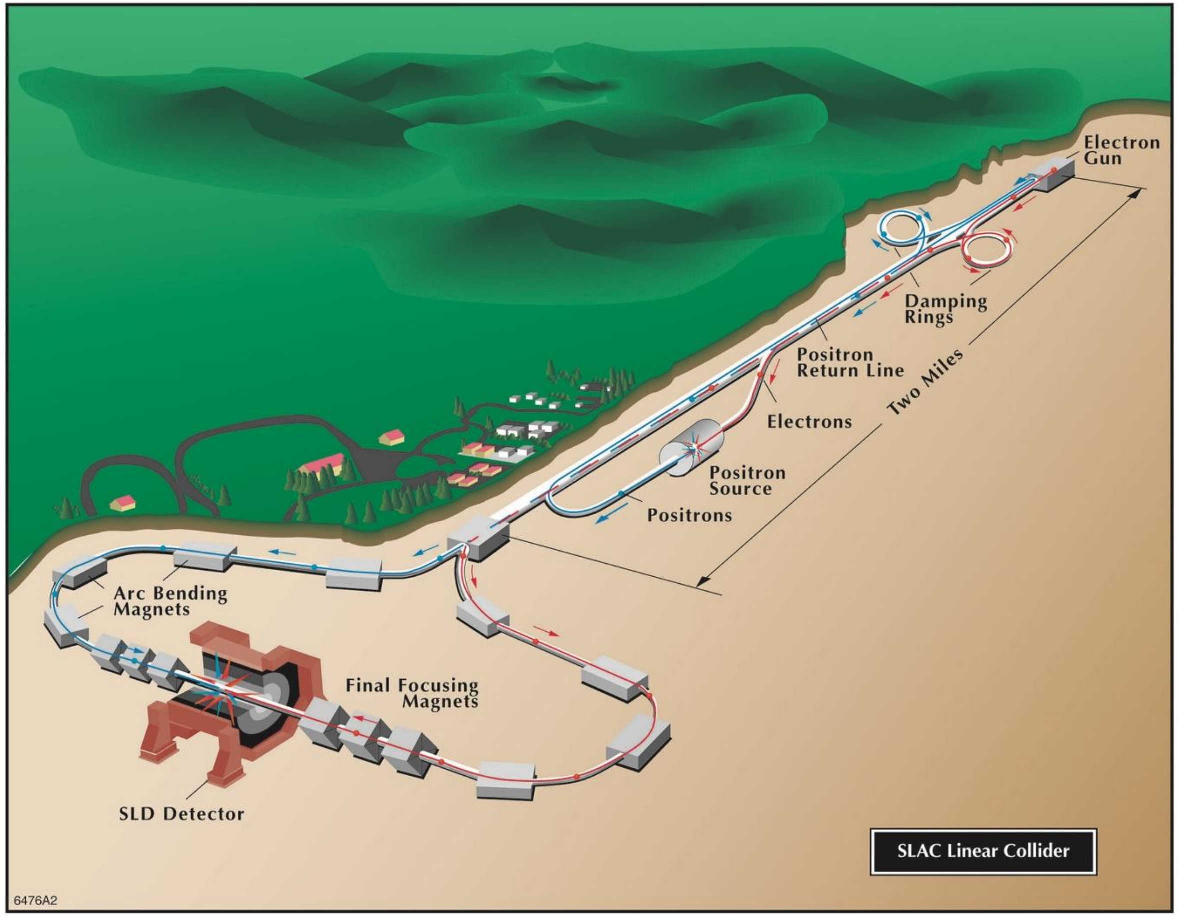
- The maximum achievable energy is constrained by B-fields technology and by Radio-Frequency cavities;
- Large accelerators … are VERY large → LHC, was limited by the space between Lake Geneva and the Jura mountains.

the maximum feasible beam energy available for high-energy physics experiments is constrained or limited by the geographical boundary conditions of the region (and by geological conditions).

SLAC Linear e^+e^- Accelerator at SLAC

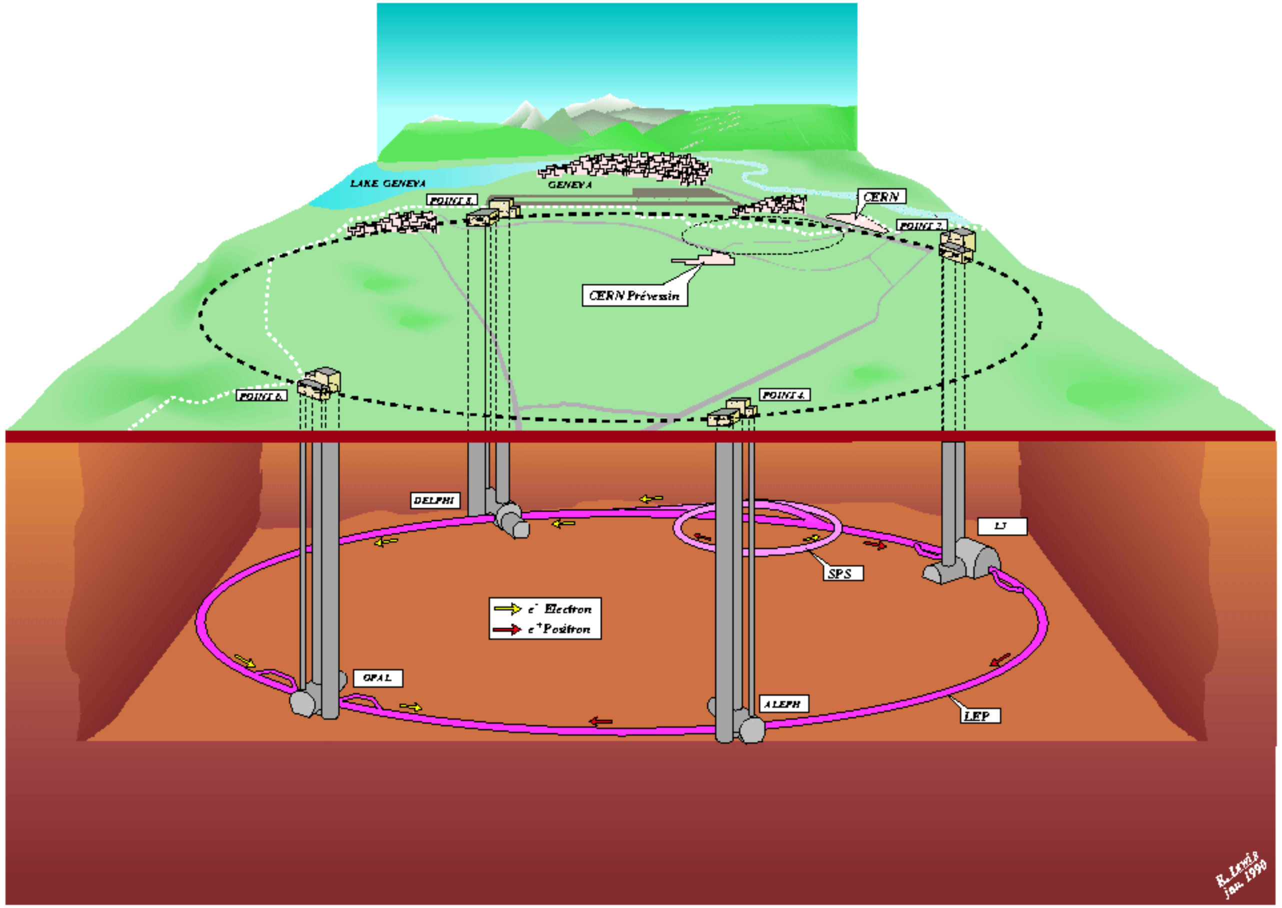
Stanford Linear Accelerator Center

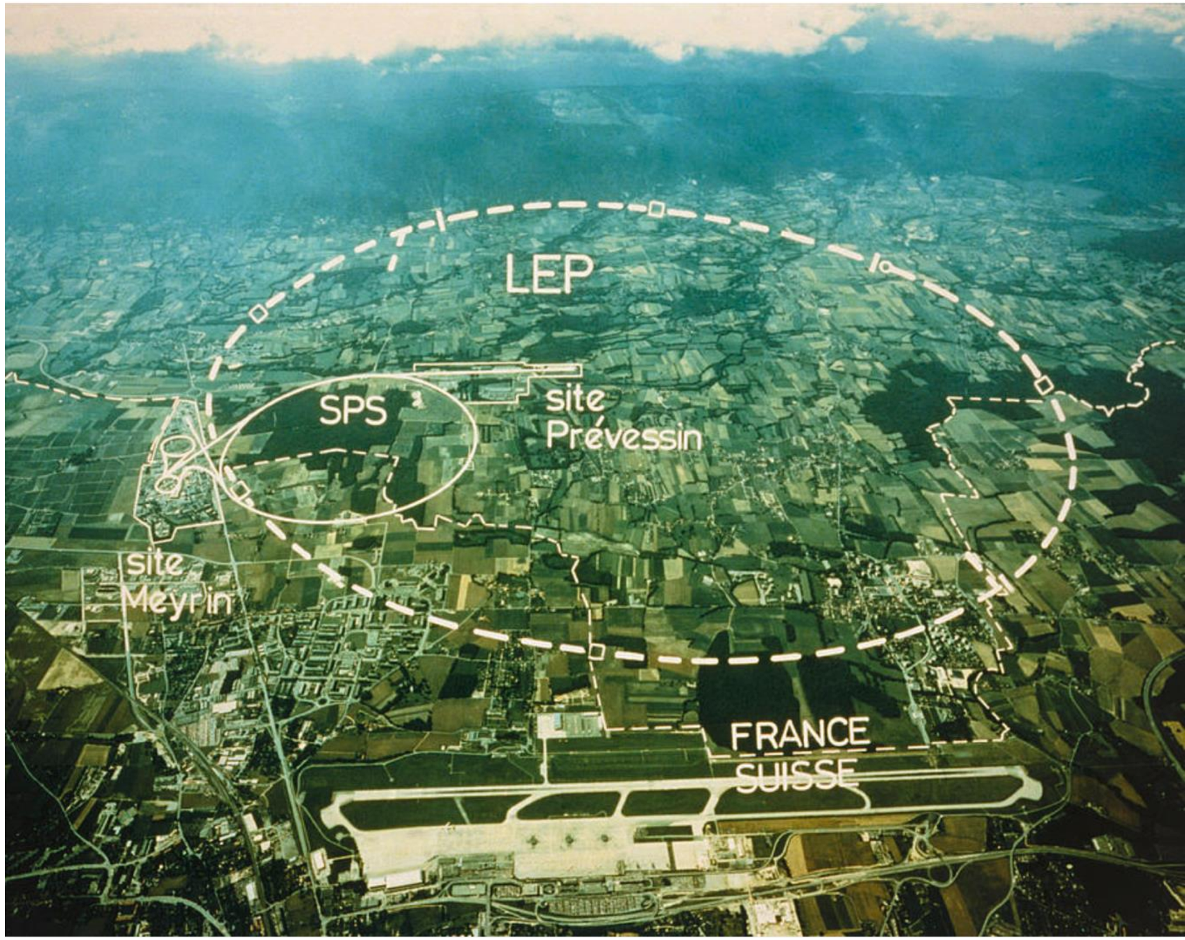
a 3.2 kilometres linear accelerator constructed in 1966 that could accelerate e^+e^- to energies of 50 GeV.



LargeElectronPositron (LEP) Collider at CERN

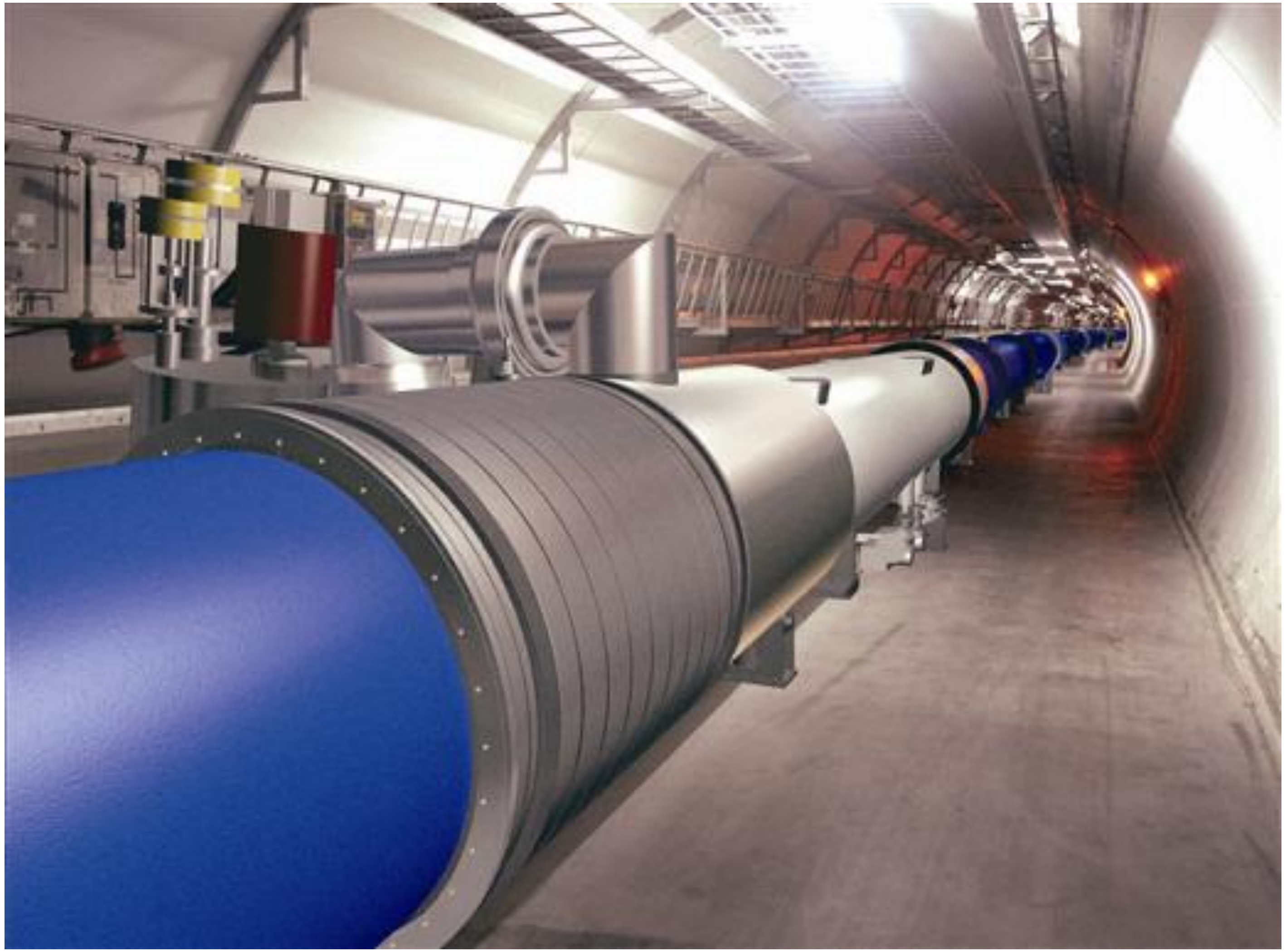
LEP collided electrons with positrons at energies that reached 209 GeV. It was a circular collider with a circumference of 27 kilometres built in a tunnel roughly 100 m (300 ft) underground and passing through Switzerland and France. LEP was used from 1989 until 2000.

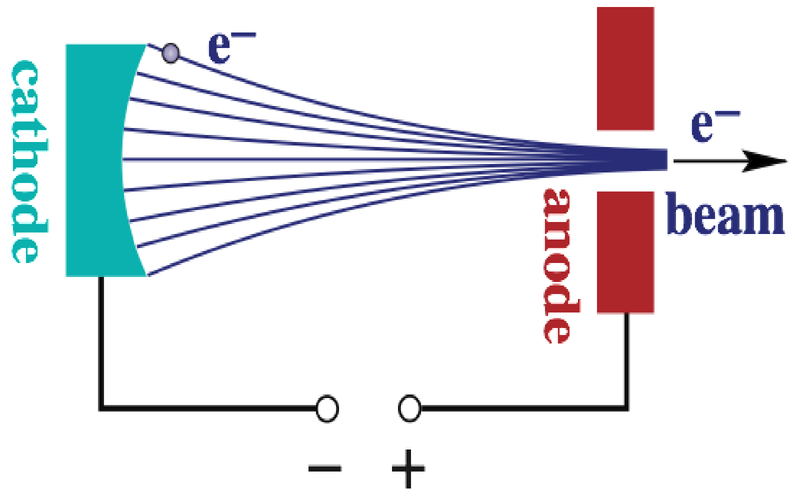


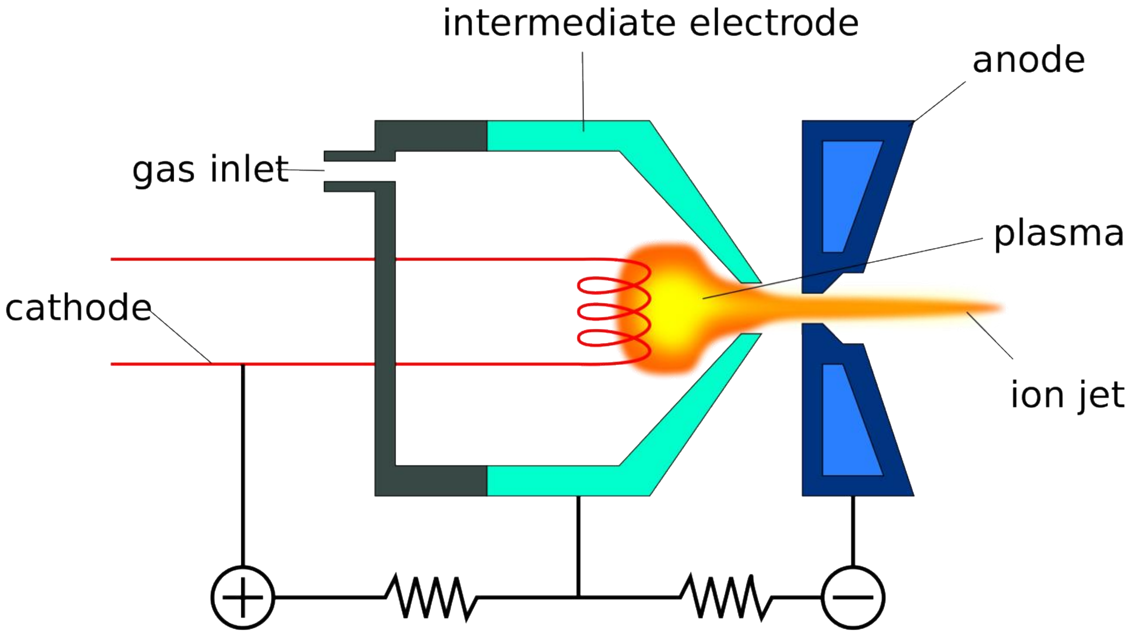


Inventory of Accelerator's Components

- Particle sources (electrons, protons.. Ions)
- Orbit creation and maintenance (magnets, dipoles & quadrupoles)
- Acceleration (RF cavities)
- Monitoring Units
- Dumps



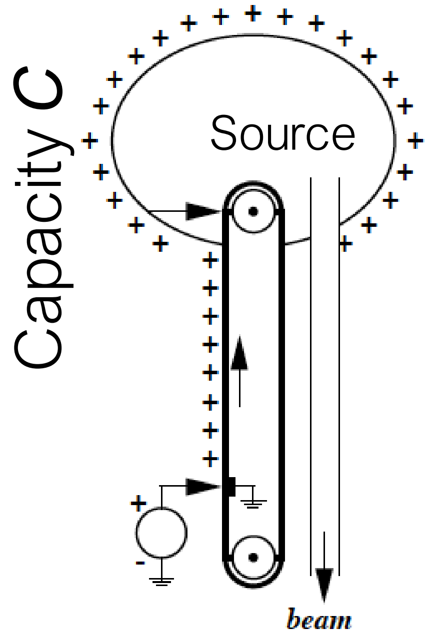




Electron source is based on the thermoionic effect of a filament heated to high temperature. Electrons emitted by the filament are accelerated by an electric field and collimated through a small window.

Similarly a source of positive ions is realised by using a combined effect of an electric field that accelerates electrons emitted by the filament and a magnetic field that makes them spiralise. The gas filling the source is ionised by the electrons and positive ions (atoms with stripped electrons) are accelerated by an electric field and emerge from a small window.

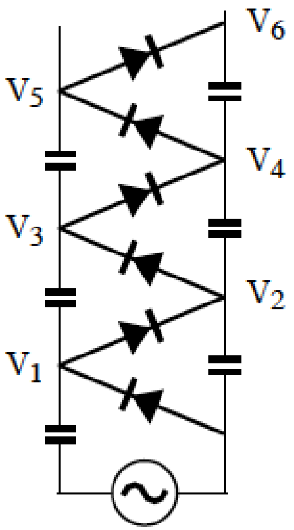
History: DC to AC Accelerators (~1930)



Van de Graaff designed a DC accelerator that used a mechanical transport system to carry charges, sprayed on a belt or chain, to a high-voltage terminal.

$$\Delta V = \int i(t)\, dt / C$$

high-voltage breakdown (discharges) limits the maximum energy.
Complex devices, can reach is of the order of ~10 MeV and currents up to ~10μA of ions.

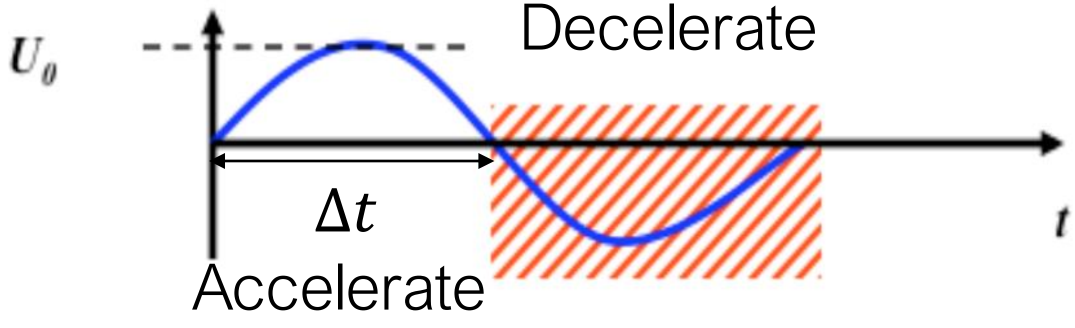


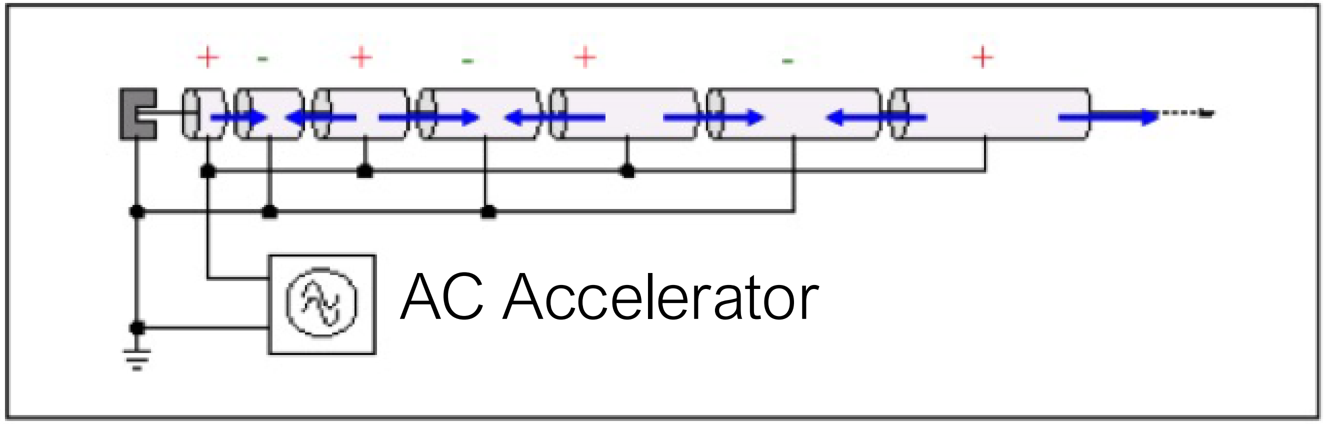
Cockcroft and Walton: acceleration mechanism based on a rectifier circuit, diodes and capacitors.

→ a multiple of the relatively small applied AC voltage.
The highest voltage these accelerators can reach is of the order of ~5 MeV and currents up to ~20μA of ions.

A Change of Paradigma: AC Accelerators (~1930)

Widerøe in ~1930 → rectify the AC voltage: a series of acceleration electrodes in an alternating manner to the output of an AC supply.





→ make particles see the accelerating field only

In principle, this device can produce a multiple of the acceleration voltage, if the particles are shielded from the decelerating field (negative half-wave of the AC voltage).

The energy gain after the n_{th} step is

$$E_n = n \cdot q \cdot U_0 \cdot \sin(\psi_s)$$

- n is the acceleration step,
- q the charge of the particle
- U_0 the applied voltage per gap
- Ψ_s the phase between the particle and the changing AC voltage.

Design of the Accelerating Structure

The duration of the accelerating phase is half of the applied frequency ($\tau_{rf} = {}^{1}/_{\nu_{rf}}$)

$$\Delta t = \frac{\tau_{rf}}{2}$$

defining the length of the n$_{th}$ drift tube

$$l_n = v_n \frac{\tau_{rf}}{2} \quad (v_n = velocity)$$

Since the kinetic energy is

$$E_{kin} = \frac{mv^2}{2}$$

We get (remember $E_n = n \cdot q \cdot U_0 \cdot \sin(\Psi_s)$ and $\tau_{rf} = {}^{1}/_{\nu_{rf}}$)

$$l_n = \frac{1}{\nu_{rf}} \cdot \sqrt{\frac{nqU_0 \sin(\Psi_s)}{2m}}$$

The accelerator dimensions CANNOT be too large

- When m ↑ then l_n ↓This approach is OK for ~relatively low energy beams, ~10 MeV. The CERN Linac produces 50 MeV protons (corresponding to β = 0.31)
- The length of the drift tube increases with increasing step
- When the requested energy increases the dimensions become impractical

→ need to go to circular machines → colliders

Today: Circular Accelerators

Limits due to the available surface → circular accelerators: the same accelerating units are used at each round every time. One can do it using electric or magnetic fields.

The Lorenz force will have to compensate exactly the centrifugal force.

Electric or magnetic fields?

$$F = q \cdot (E + v \times B)$$

E field is not amplified! → not very effective

B field is amplified by "v" (for relativistic particles this is very large) → very effective

Use only the B-part of the Lorenz force, use protons, define ρ the radius of your accelerator, $p = \gamma m v$

$$F_{Lorentz} = e \cdot v \cdot B$$

$$F_{centrifugal} = \frac{\gamma m v^2}{\rho}$$

⇒

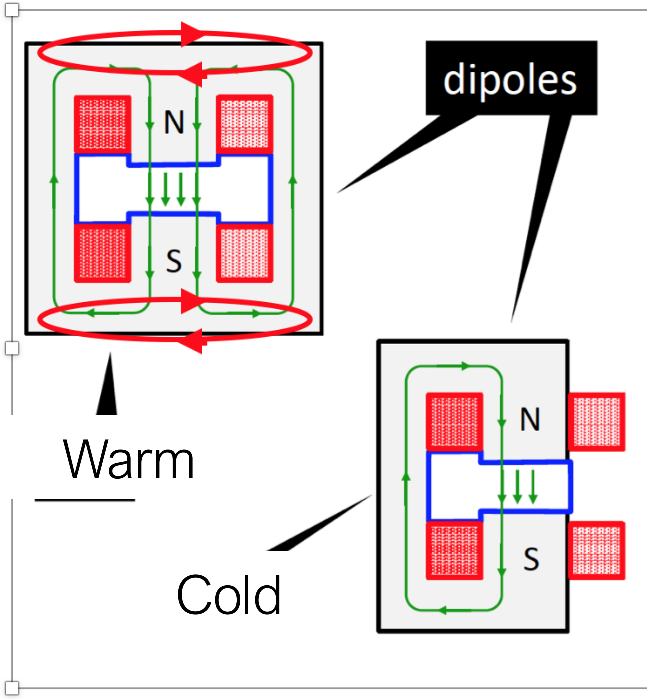
$$\frac{p}{e} = B \cdot \rho$$

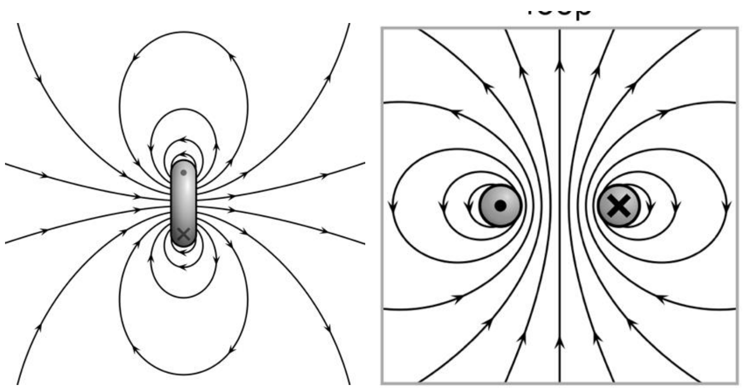
$$B(T)\rho(m) = p(GeV/c) \cdot 3.33$$

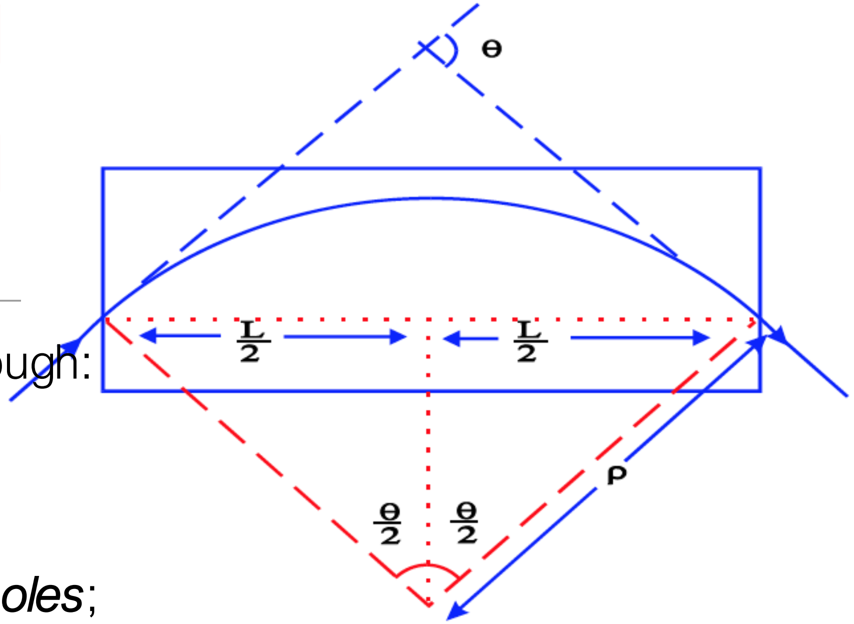
Keeping Particles on a Circular Trajectory

The closed orbits inside which particles travel is called '*the design orbit*' or '*golden orbit*'.

- ➢ Use dipoles
- ➢ a sequence of dipoles are used (not one)
- ➢ → the nominal orbit is not really a circumference but rather a polygon.
- ➢ The bending occurs inside the dipole, before and after the trajectory is a straight line.







Bending (→ keeping the beam in a HORIZONTAL ~circular orbit) is not enough:

1. Accelerate particles → ***RF cavities***;
2. compensate for possible energy losses along the orbit → ***RF cavities***;
3. Focus the beam to a limited size to maximize the luminosity → ***Quadrupoles***;
4. Equip your accelerator with systems to dump in a controlled/uncontrolled manner your beam(s) → ***Dumps***;

A Polygonal Accelerator

To have a closed orbit the total bending angle has to be equal to 2π.
If α is the bending angle of one dipole then (Use p=βγmc and p/E=Bρ)

$$B = \frac{2\pi \cdot 7000 \cdot 10^9 \text{ eV}}{1232 \cdot 15 \text{ m} \cdot 2.99792 \cdot 10^8 \text{ m s}^{-1}}$$

$$\alpha = \frac{ds}{\rho} = \frac{B \cdot ds}{B \cdot \rho} \rightarrow \frac{\int Bdl}{B \cdot \rho} = \frac{\int Bdl}{p/e} = 2\pi \rightarrow \int Bdl = 2\pi \cdot p/E$$

to get

At LHC $$\int Bdl = 1232 \cdot 15m \cdot B_{dipole} \rightarrow B_{dipole} = \frac{2\pi \cdot 7TeV}{1232 \cdot 15m \cdot c} = 8.33T$$

1232 x 15 m =18.5 Km
LHC circumference ~ 27 Km

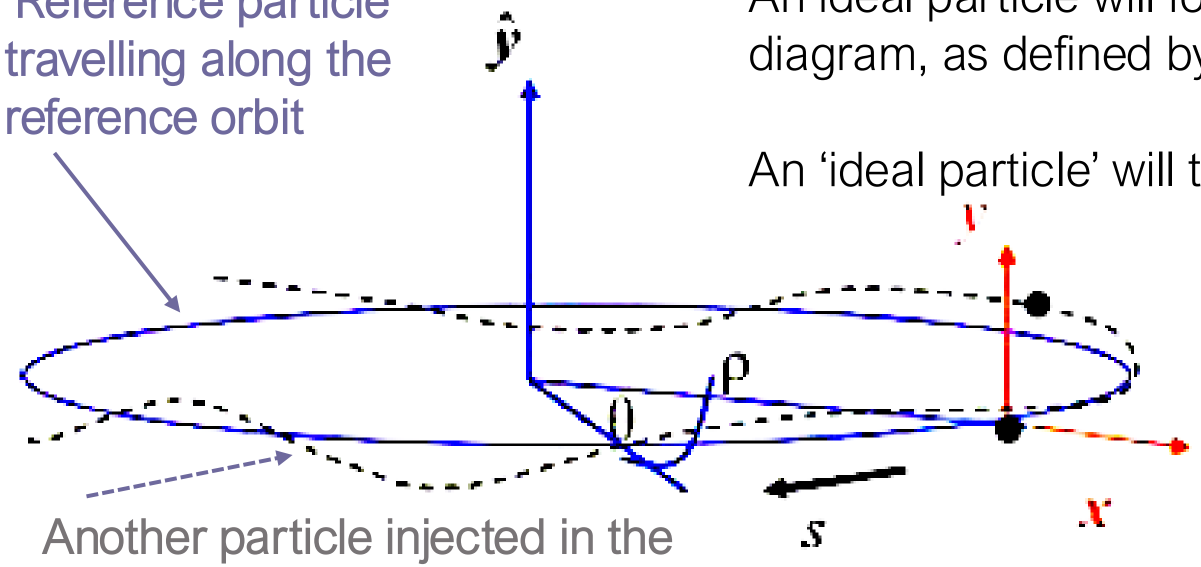
Dipoles in recent hadronic accelerators

Accelerator	Accelerator type	Beam Energy (TeV)	Circumference (Km)	No. of dipoles	Dipole Length (m)	Magnetic field (T)
Hera	ep	0.030/0.92	6.3	396/416	9.2/8.8	0.274/5
Tevatron	$p\bar{p}$	0.980	6.3	774	6.1	4.4
RHIC	pp	0.255	3.8	204	9.5	3.5
LHC	pp	4	26.7	1232	14.3	8.3
LHC	pp	7	26.7	1232	14.3	8.3

The Design Orbit and Betatron Oscillations

An ideal particle will follow the design orbit that is represented by the circle in the diagram, as defined by the network of dipoles.

‘Reference particle’ travelling along the reference orbit



Another particle injected in the accelerator with slightly wrong initial conditions

An ‘ideal particle’ will travel exactly along the circle (if injected exactly at the initial conditions used in the design of the accelerator, in practice injected in the horizontal plane, tangent to the circle). Solid line

Any other particle with **different initial conditions** (or tiny **differences in the B field** of dipoles) will remain inside the accelerator, close to the ‘circular’ path defined by the dipole fields. However it will do harmonic oscillations in both transverse planes. Dotted line

In practice it will spiral around the design orbit.

→ *Betatron Oscillations*

A beam is normally populated by a large number of particles, → the overall amplitude of betatron oscillations will define the beam size.

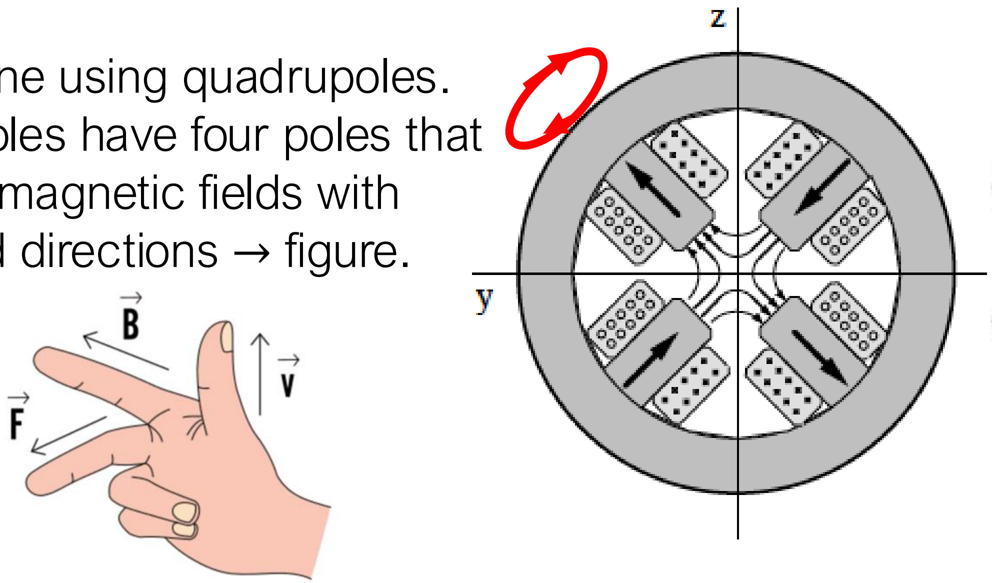
→ Restoring forces are needed

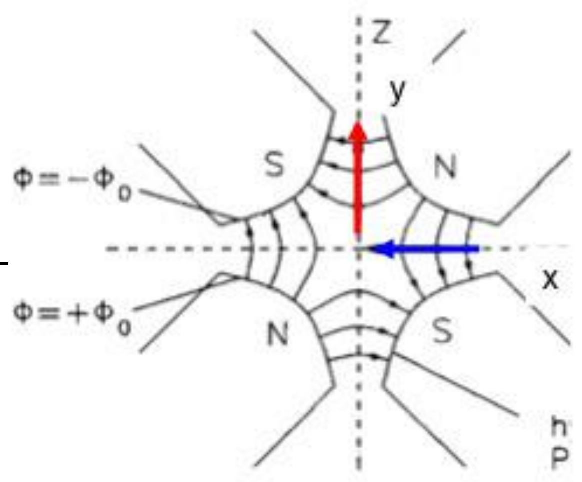
Focusing Beams

- Beams have to be extremely narrow, fractions of mm; the two beams have to meet at interactions point with great precision;
- In large colliders there is a large number of magnetic units (O 1000/2000). Small imperfections and defects (position, voltages…) may affect the particle trajectory and displace particles from the 'golden orbit';
- In a collider beams are kept for many hours and the risk of deviations from the reference orbit increases with time.

→ This implies correction mechanisms capable of restoring the trajectory of particles travelling in the wrong direction / position;

This is done using quadrupoles. Quadrupoles have four poles that generate magnetic fields with alternated directions → figure.

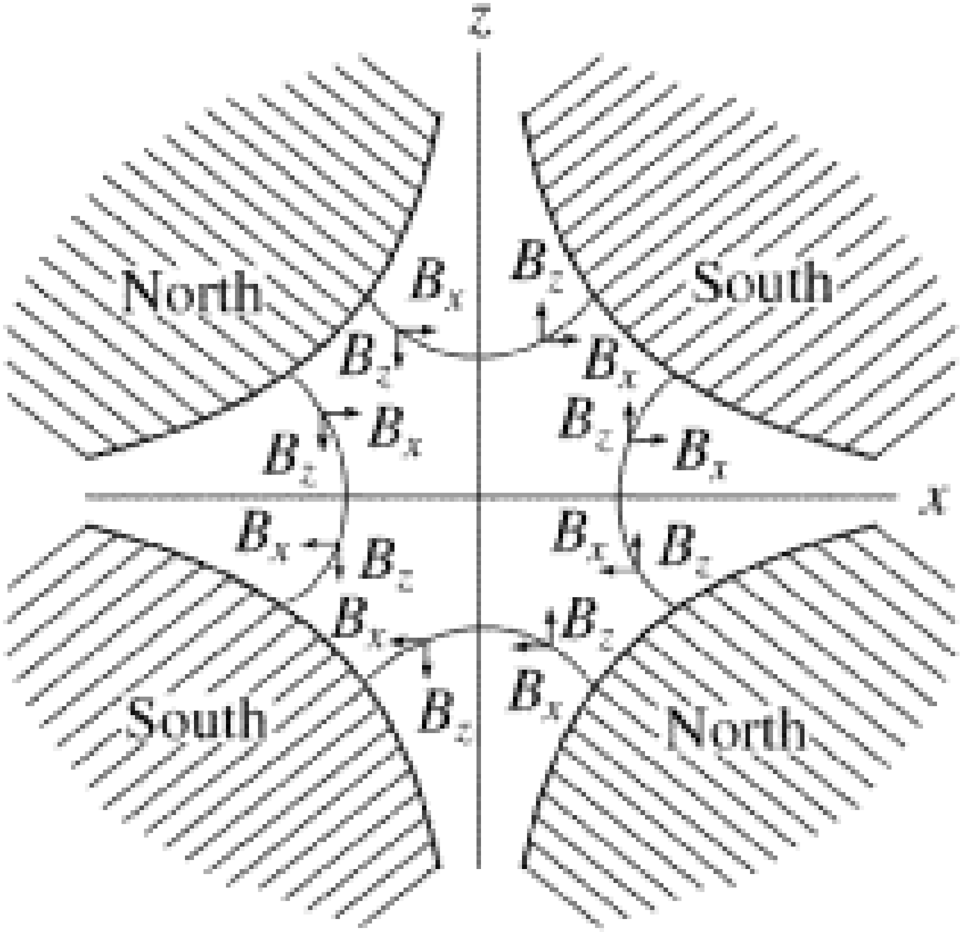


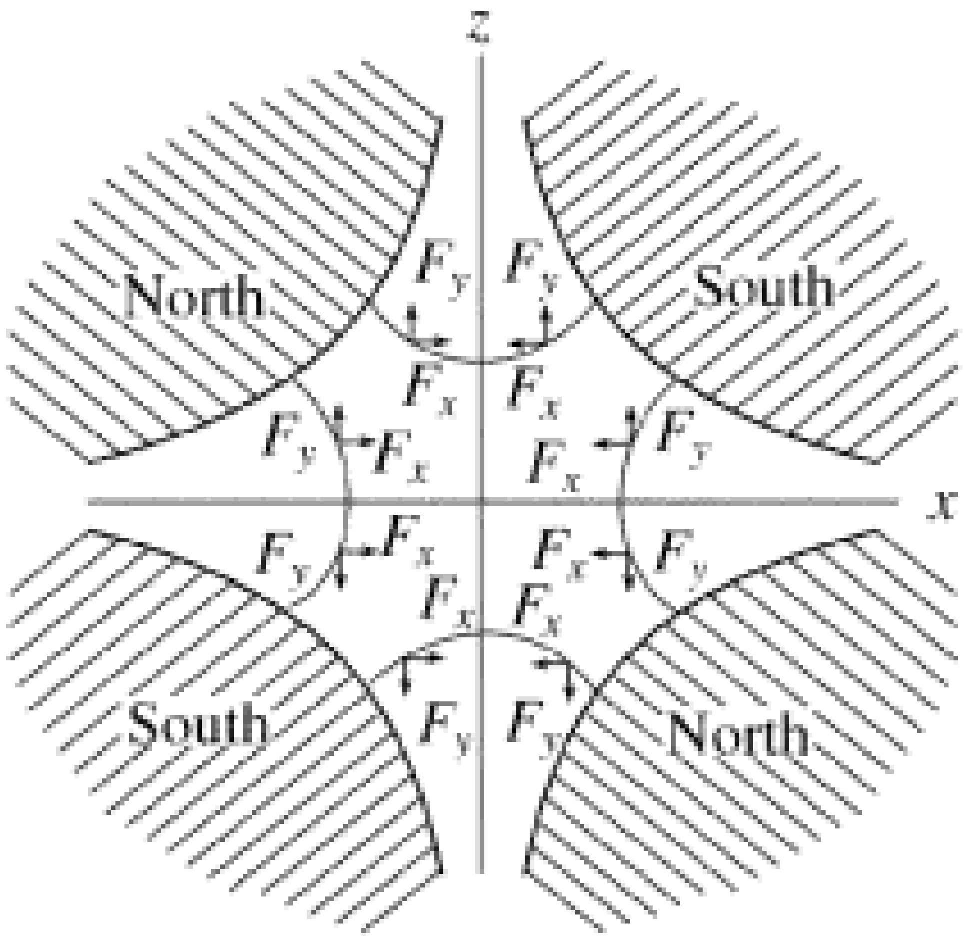


Particles travelling at the centre of the quadrupole do not experience any field and are unaffected.

Particles away from the axis in x are focused

Particles away from the axis in y are de-focused


z
x
North
South
South
North
B_x
B_z


z
x
North
South
South
North
F_x
F_y

Focusing Properties of Quadrupoles

The quadrupole field depends linearly on the transverse position:

$$B_x = g \cdot y$$
$$B_y = g \cdot x$$

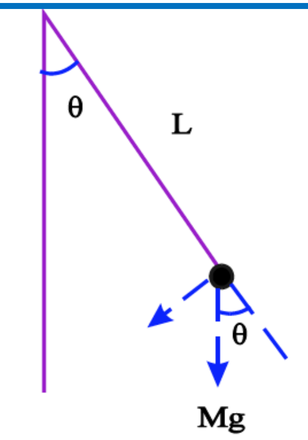
Quadrupoles

$$\frac{p}{e} = B \cdot \rho$$

The constant g characterises the focusing strength of the quadrupole. A derived constant k is introduced: focusing strength of the quadrupole normalised to the particle momentum

$$k = \frac{g}{p/e} = \frac{g}{B \cdot \rho}$$

Dipoles



$$\ddot{\theta} + \frac{g}{l}\theta = 0$$

The situation in the orbit plane (horizonal where the acceleration takes place) and in the vertical are a bit different, Equations describing the motion in the two planes are identical to those of the pendulum.

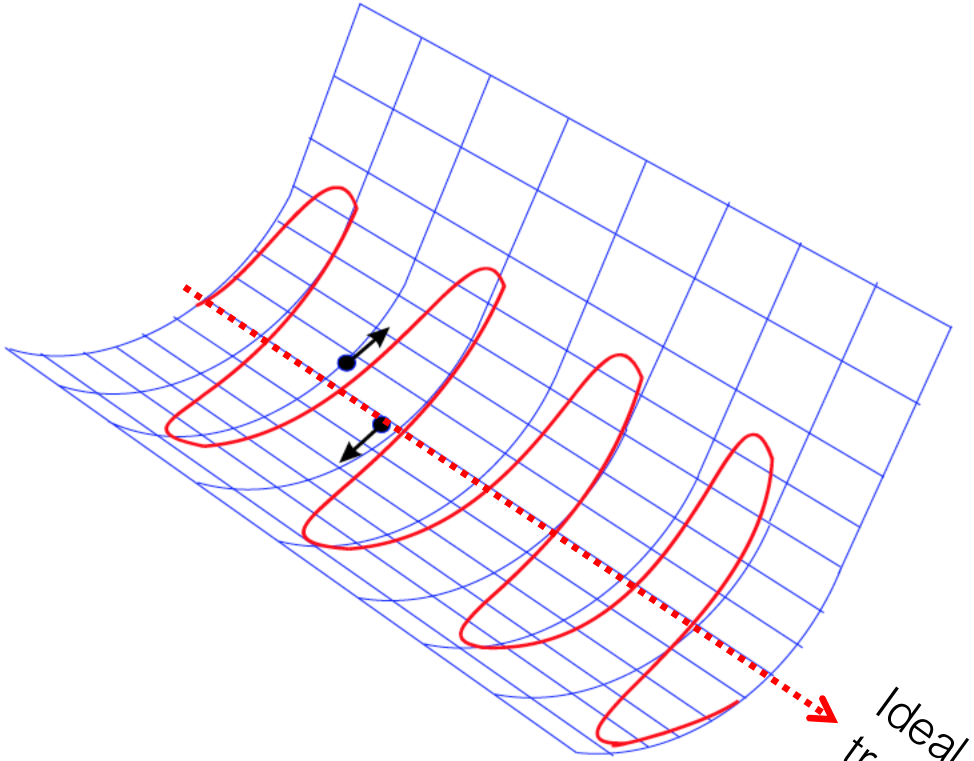
$$x'' + x \cdot \left(\frac{1}{\rho^2} + k\right) = 0, \qquad y'' - y \cdot k = 0.$$

Equations in the two planes are identical but for a 'weak focusing' term $\frac{1}{\rho^2}$ present only in the horizontal plane

Problem now is defining a set of dipoles and quadrupoles that guarantees an overall focusing effect and keeps the size of the beam constant and small (to increase luminosity, see later)

A Ball in a Gutter

Constant radius of curvature ρ



Machine Optics, Focusing Quadrupoles

In the horizontal plane the solution to the equation $x'' + x \cdot \left(\frac{1}{\rho^2} + k\right) = 0,$

x_0 *and* x'_0 are the initial conditions at the entrance of the quadrupole and $K = k - \frac{1}{\rho^2}$

Would be=0. for 0 angle

is given for
- x(s) (displacement)
- and x'(s) (angle)

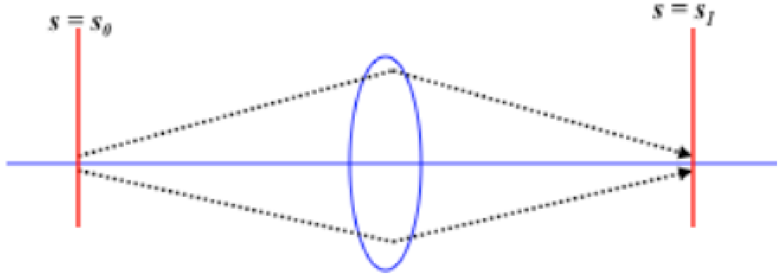
by

$$x(s) = x_0 \cdot \cos(\sqrt{|K|}\, s) + x'_0 \cdot \frac{1}{\sqrt{|K|}} \sin(\sqrt{|K|}\, s)$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \sin(\sqrt{|K|}\, s) + x'_0 \cdot \cos(\sqrt{|K|}\, s)$$

Vertical plane has the same solutions but for the fact $K = k$

Effect of a **focusing** quadrupole on the trajectory of a charged particle

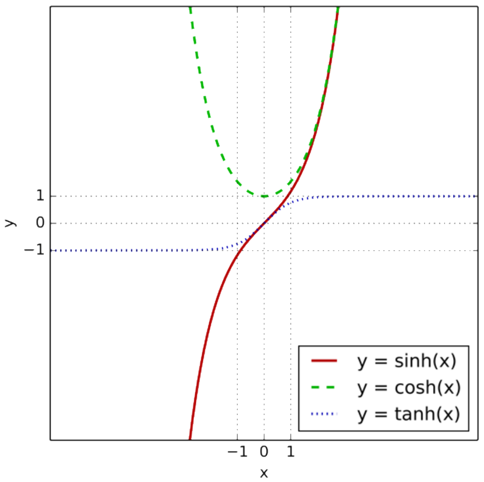


The two equations above can be written in a more compact way using matrices

$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M_{foc} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix}$$ where $M_{foc} = \begin{pmatrix} \cos(\sqrt{|K|} \cdot s & \frac{1}{\sqrt{|K|}} \cdot \sin(\sqrt{|K|} \cdot s \\ -\sqrt{|K|} \cdot \sin(\sqrt{|K|} \cdot s & \cos(\sqrt{|K|} \cdot s \end{pmatrix}$

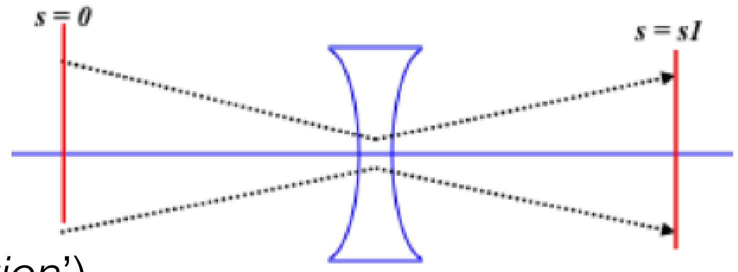
Machine Optics, Defocusing Quadrupoles

Similarly for a defocusing quadrupole an almost identical matrix equation can be written



$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M_{defoc} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad \text{where } M_{defoc} = \begin{pmatrix} \cosh(\sqrt{|K|} \cdot s) & \frac{1}{\sqrt{|K|}} \cdot \sin h(\sqrt{|K|} \cdot s) \\ \sqrt{|K|} \cdot \sinh(\sqrt{|K|} \cdot s) & \cosh(\sqrt{|K|} \cdot s) \end{pmatrix}$$

Effect of a **defocusing** quadrupole on the trajectory of a charged particle

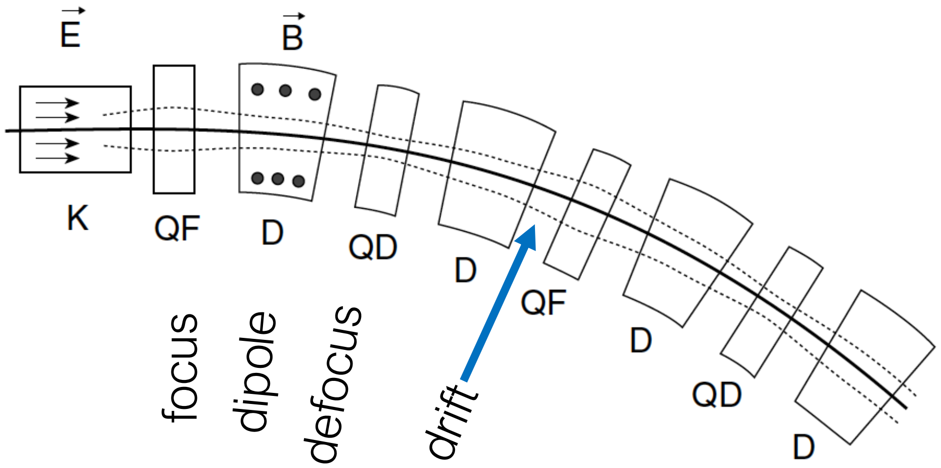


The drift in a region without magnetic field (‘*straight section*’)

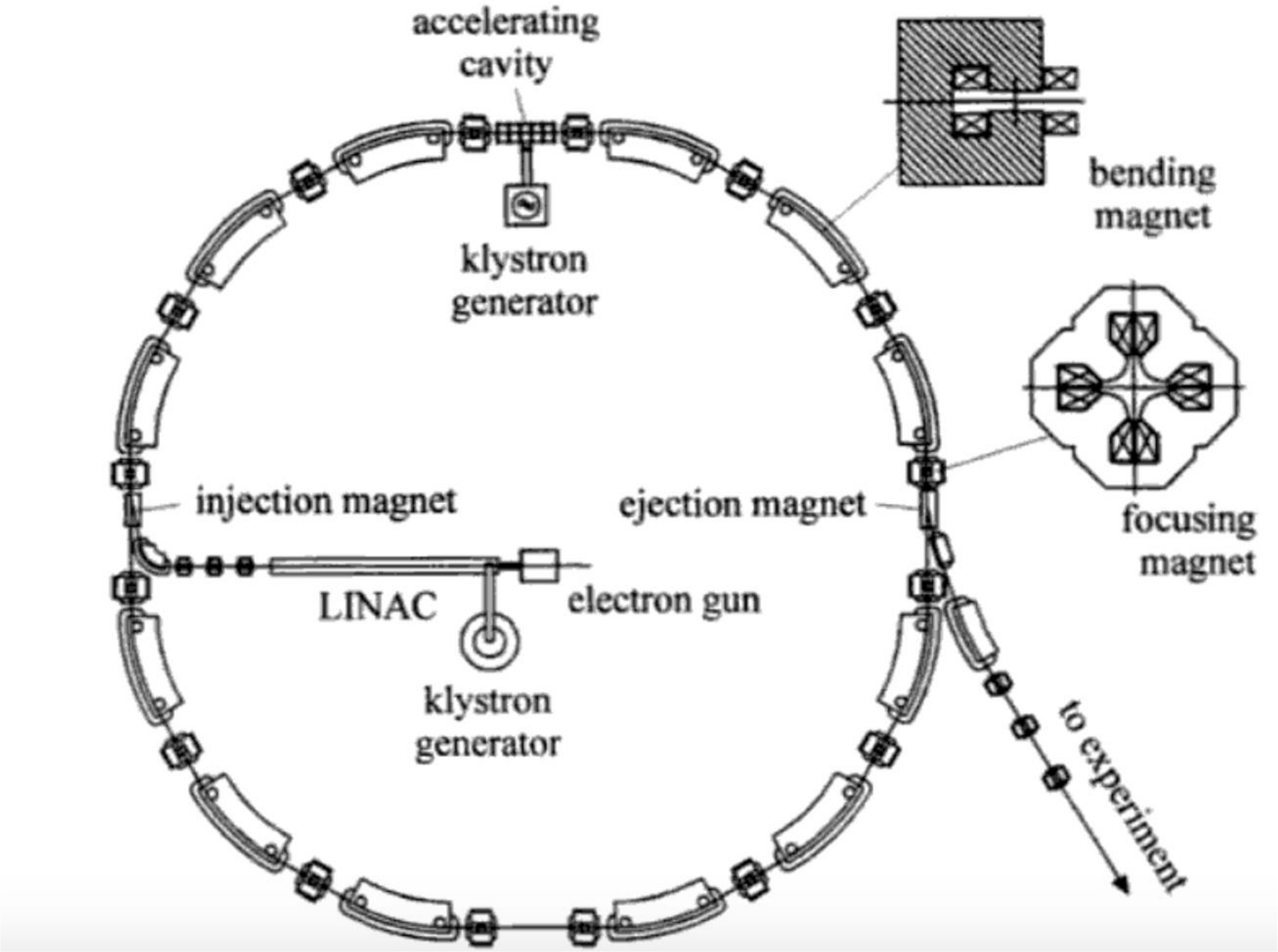
$$\begin{pmatrix} x(s) \\ x'(s) \end{pmatrix} = M_{drift} \begin{pmatrix} x_0 \\ x'_0 \end{pmatrix} \quad \text{where } M_{drift} = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix}$$

The total displacement now can be expressed as the product of matrices

$$M_{total} = M_{foc} \cdot M_{drift} \cdot M_{dipole} \cdot M_{drift} \cdot M_{defoc} \ldots\ldots$$

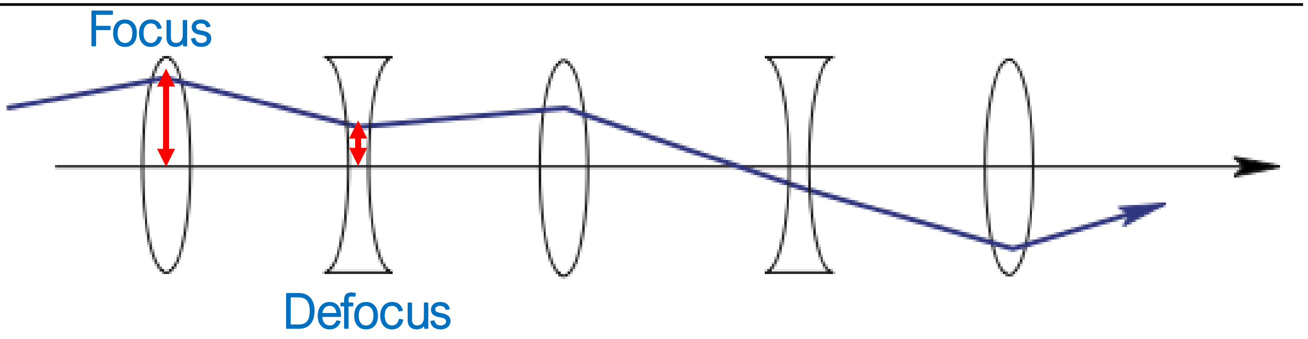


The Structure of a One Beam Accelerator



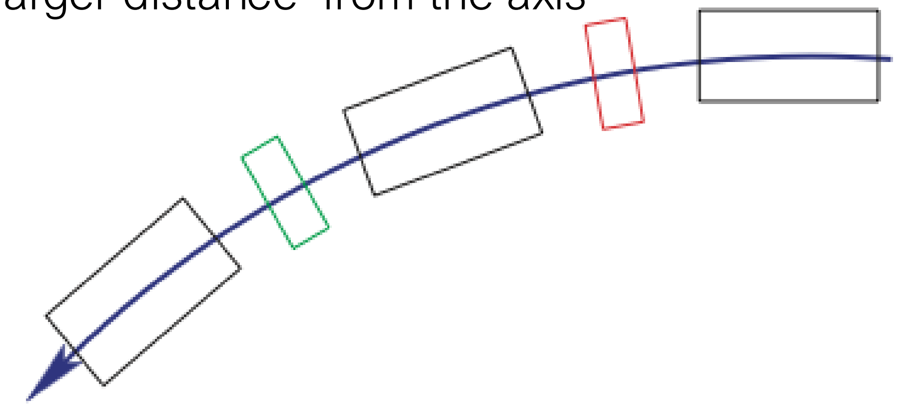
Total Focusing

An appropriate choice of focusing and defocusing quadrupoles gives a net focusing effect in both projections.



Alternating focusing and defocusing quadrupoles leads (may lead!) to an overall focusing:

- the focusing quadrupoles are, on the average, traversed at larger distance from the axis
- than the defocusing ones



Analogy with optics:

Focal length of two lenses at distance D

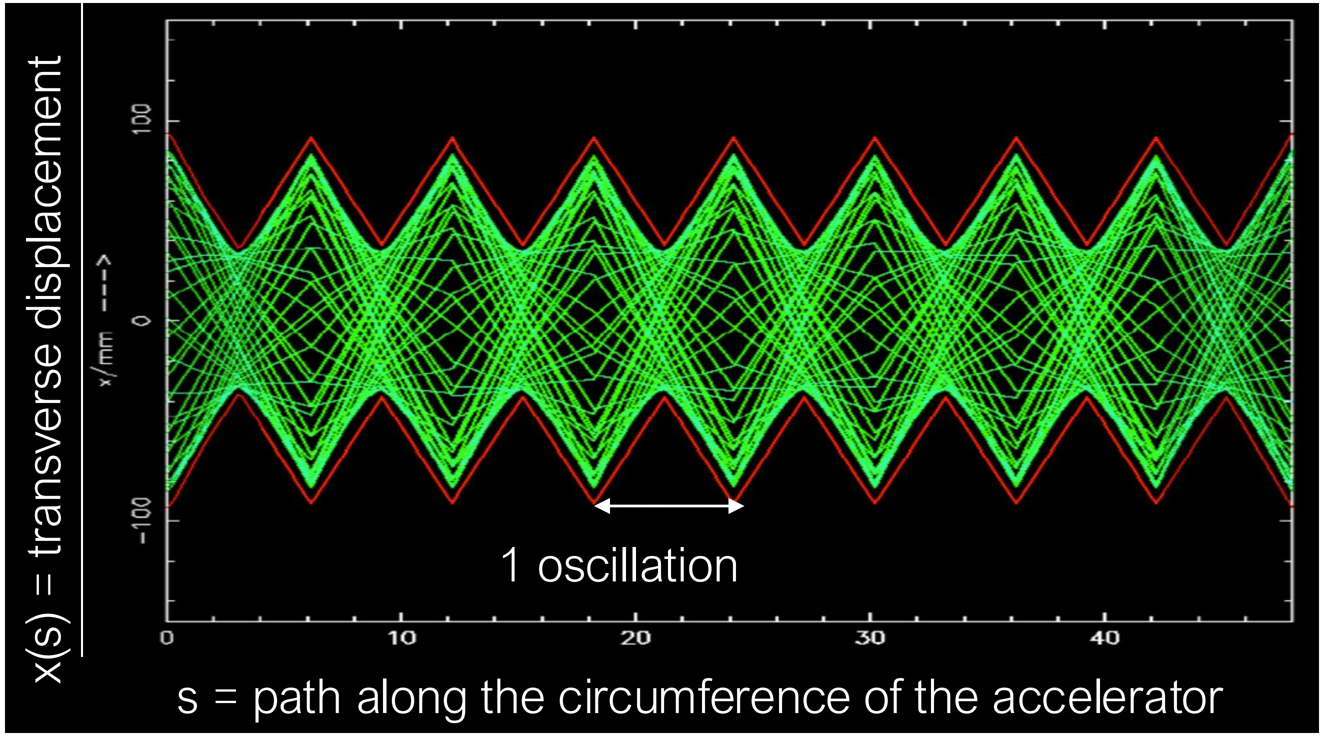
$$1/f = 1/f_1 + 1/f_2 - D/f_1 f_2$$

is overall focusing

when $f_1 = -f_2 \Rightarrow$

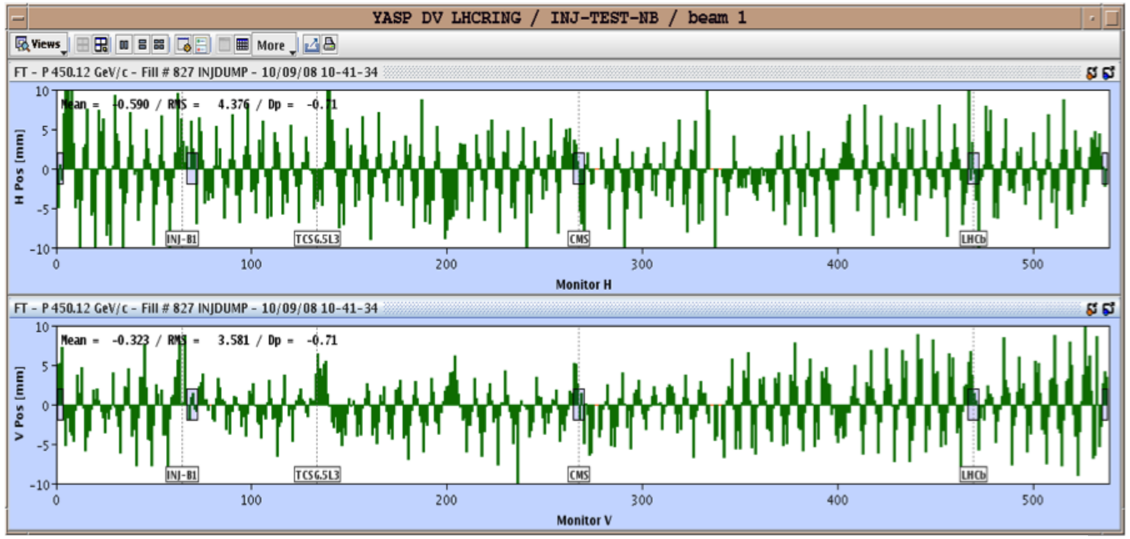
$$f = D$$

Total Focusing



- At each moment (~in each lattice element) the trajectory is an harmonic oscillation.
- Due to the different focusing or defocusing forces, the solution will be different at each location.
- **All particles experience the same fields, and their trajectories will differ only because of their different initial conditions.**
- There is an overall oscillation in both transverse planes while the particle is travelling around the ring. Its amplitude must stay well within the vacuum chamber

A Real Collider: LHC

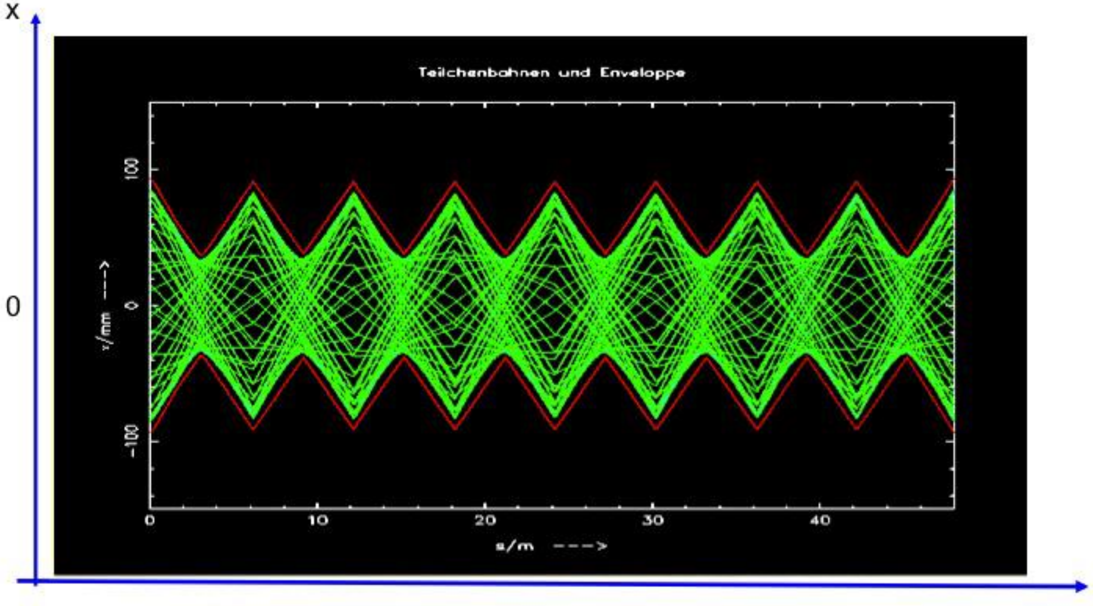


The figure on the left shows the transverse oscillations in the transverse plane of LHC as measured by the monitoring equipment in 1 turn. One important parameter is the the number of oscillations in the horizontal and in the vertical plane

The values measured are:
(in one LHC run)

Qx = 64:31 Qy = 59:32

Not an integer number → orbit changes at each turn



What is the trajectory of the particle after an arbitrary number of turns? **See later**

Circular machine: the amplitude and angle, x and x', at the end of the first turn will be the initial conditions for the second turn, and so on. After many turns the overlapping trajectories begin to form a pattern.

Fig ← beam has larger and smaller beam size but remains well-defined in its amplitude by the external focusing forces.

From Simple Oscillator to Hill's Equation

Simple Harmonic Motion:

$$x'' + x \cdot \left(\frac{1}{\rho^2} + k\right) = 0$$

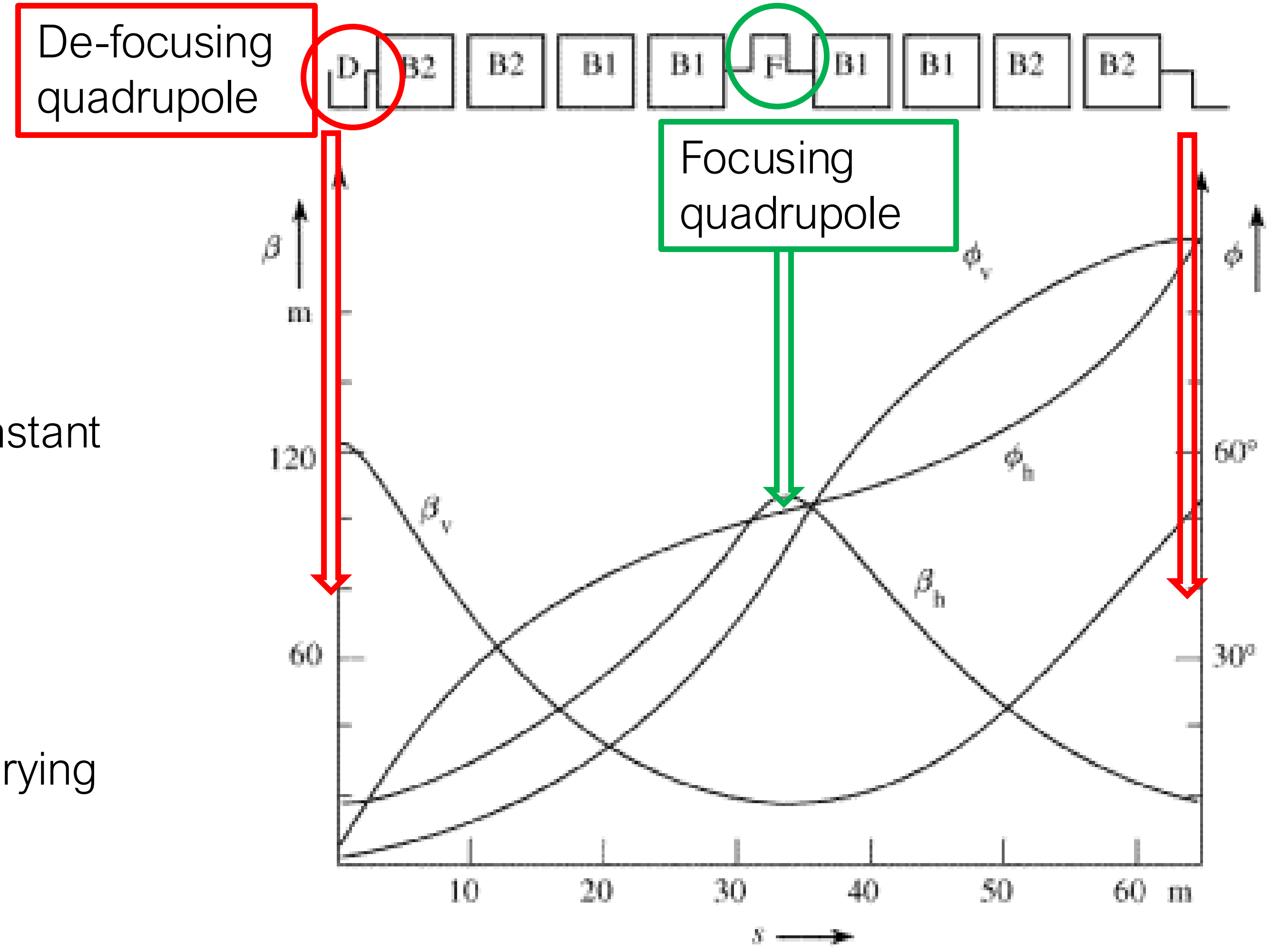
restoring force $\boldsymbol{k}$ and focusing term $\boldsymbol{\frac{1}{\rho^2}}$ are constant

Hill's Equation:

$$x'' + x \cdot \left(\frac{1}{\rho(s)^2} + k(s)\right) = 0$$

restoring force $k(s)$ and focusing term $\frac{1}{\rho(s)^2}$ varying along the trajectory s

$$x(s) = \sqrt{\epsilon\, \beta(s)} \cos(\mu(s) + \phi)$$

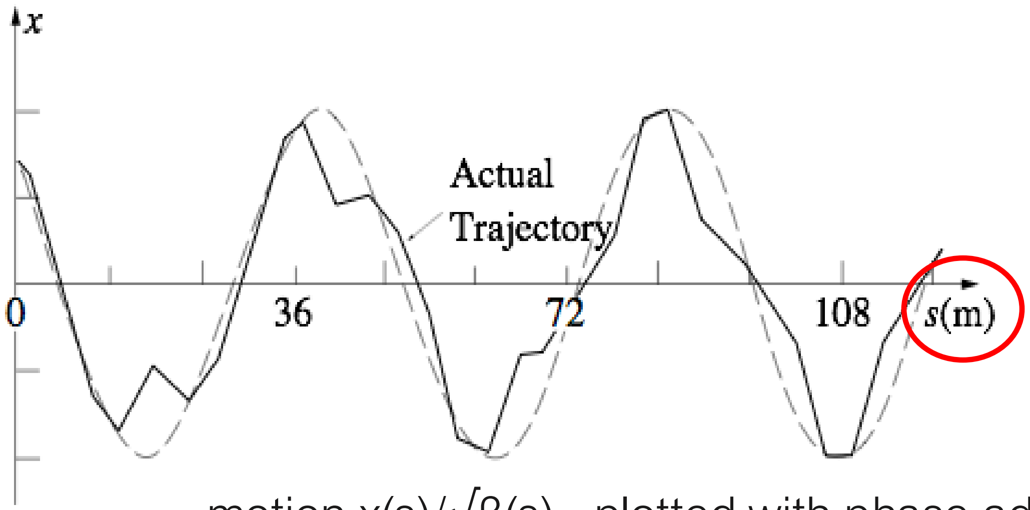


Planetary Motion → Equation of beam particles

The equation of motion of particles in a ring (with bending fields → dipoles) and quadrupoles (field gradients ∝ ∂B/∂r) which describe the motion of particles in a beam is given by (in both transverse planes),

$$x''(s) + k(s)\, x(s) = 0,$$

(known as Mathieu-Hill equation) and derived in 1801 to describe the planetary motion



motion x(s)/√β(s) plotted with phase advance normalised coordinates - becomes simple cos

Would be=0. for 0 angle

$$x(s) = x_0 \cdot \cos\left(\sqrt{|K|}\, s\right) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\, s\right),$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \sin\left(\sqrt{|K|}\, s\right) + x_0' \cdot \cos\left(\sqrt{|K|}\, s\right)$$

Variable restoring forces)

Solution : $x(s) = \sqrt{\epsilon\, \beta(s)} \cos(\mu(s) + \phi)$

s is the position along the ring

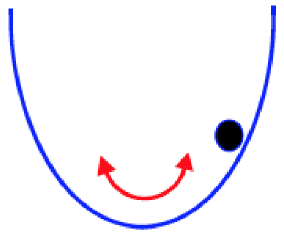
A Ball in a Gutter with Variable Shape

s is the position along the ring

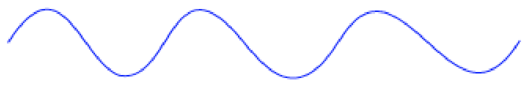
$$x(s) = x_0 \cos\left(\sqrt{|K|}\, s\right) + x_0' \cdot \frac{1}{\sqrt{|K|}} \sin\left(\sqrt{|K|}\, s\right),$$

$$x'(s) = -x_0 \cdot \sqrt{|K|} \sin\left(\sqrt{|K|}\, s\right) + x_0' \cdot \cos\left(\sqrt{|K|}\, s\right)$$

$x_0, x_0' = initial\ conditions\ (position\ \&\ angle)$
$K = 'constant\ radius'\ gutter, restoring\ force$

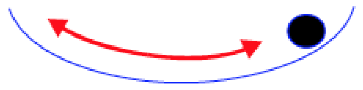


1) Amplitude small
2) phase of betatron oscillations advances rapidly with s.

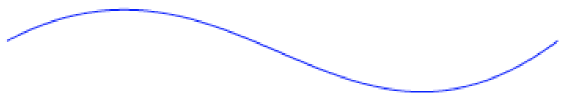


$$x(s) = \sqrt{\epsilon\, \beta(s)} \cos(\mu(s) + \phi)$$

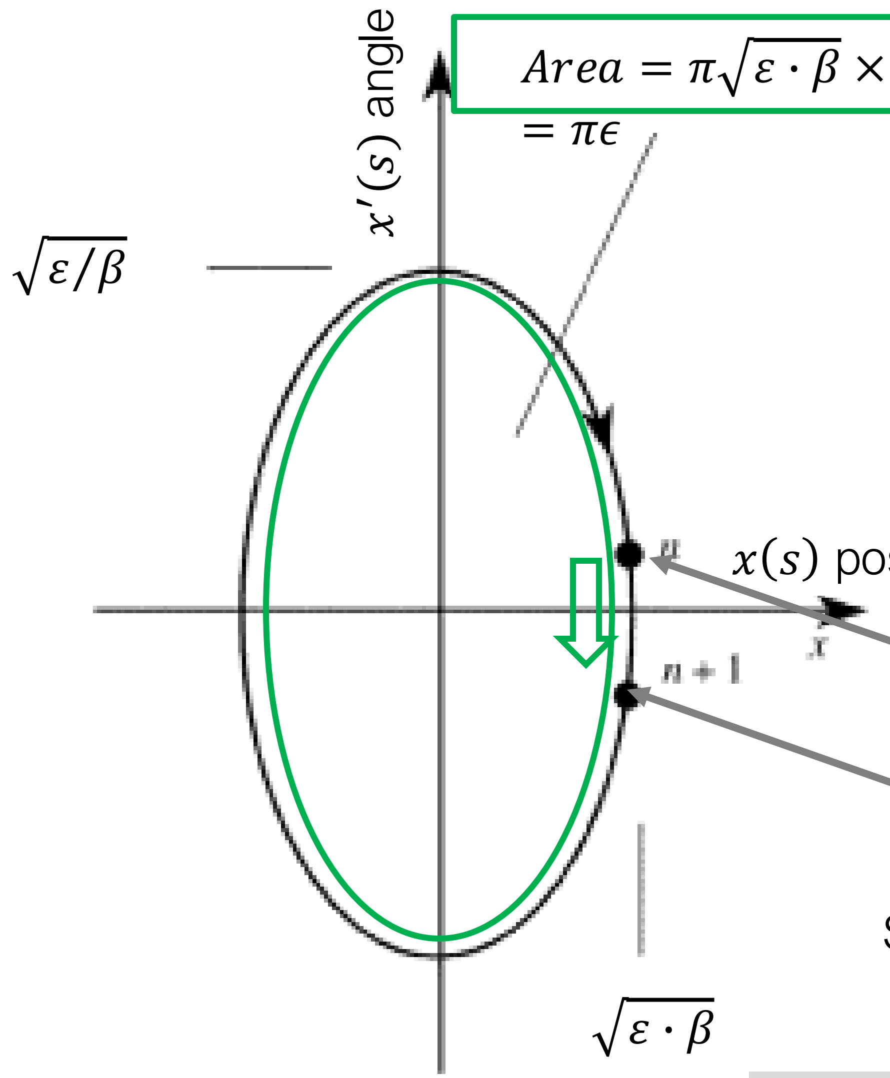
- ε, ϕ *depend on initial conditions*
- $\beta(s) =$ *amplitude modulation variable restoring force*
- $\mu(s) =$ *phase advance, also depending on restoring force*



1) Amplitude large
2) phase of betatron oscillations advances slowly with s.



The ε (emittance) Ellipse



$x(s)$ transverse position
$x'(s)$ angle

$$x(s) = \sqrt{\beta(s) \cdot \epsilon}\,\cos(\mu(s) + \phi)$$

$$x'(s) = -\sqrt{\frac{\epsilon}{\beta(s)}}\sin[\mu(s) + \phi] + \frac{\beta'(s)}{2}\sqrt{\frac{\epsilon}{\beta(s)}}\cos(\mu(s) + \phi)$$

Let's study the case when $\beta'(s) = 0$.
→ get the ellipse in the left

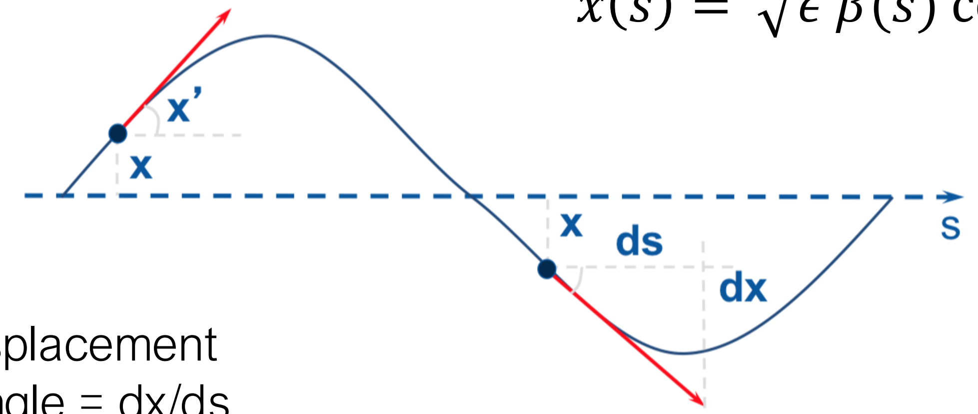
Start of trajectory (s) @ $turn\ n$

Start of trajectory (s) @ $turn\ n + 1$

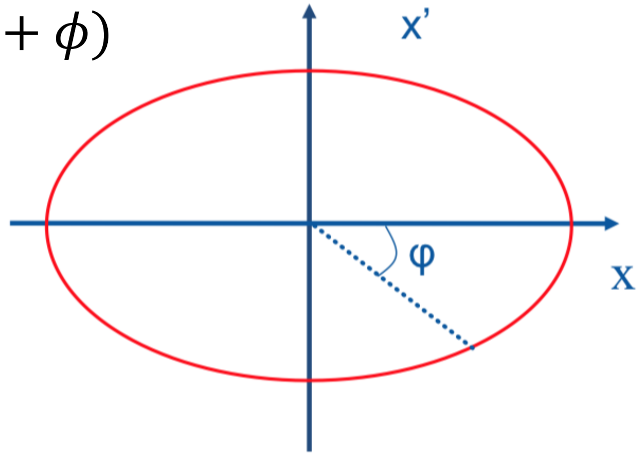
Small emittance → beam has a small size and a small divergence

Betatron Oscillations & Emittance

$$x(s) = \sqrt{\epsilon\,\beta(s)}\,\cos(\mu(s) + \phi)$$



x = displacement
x' = angle = dx/ds

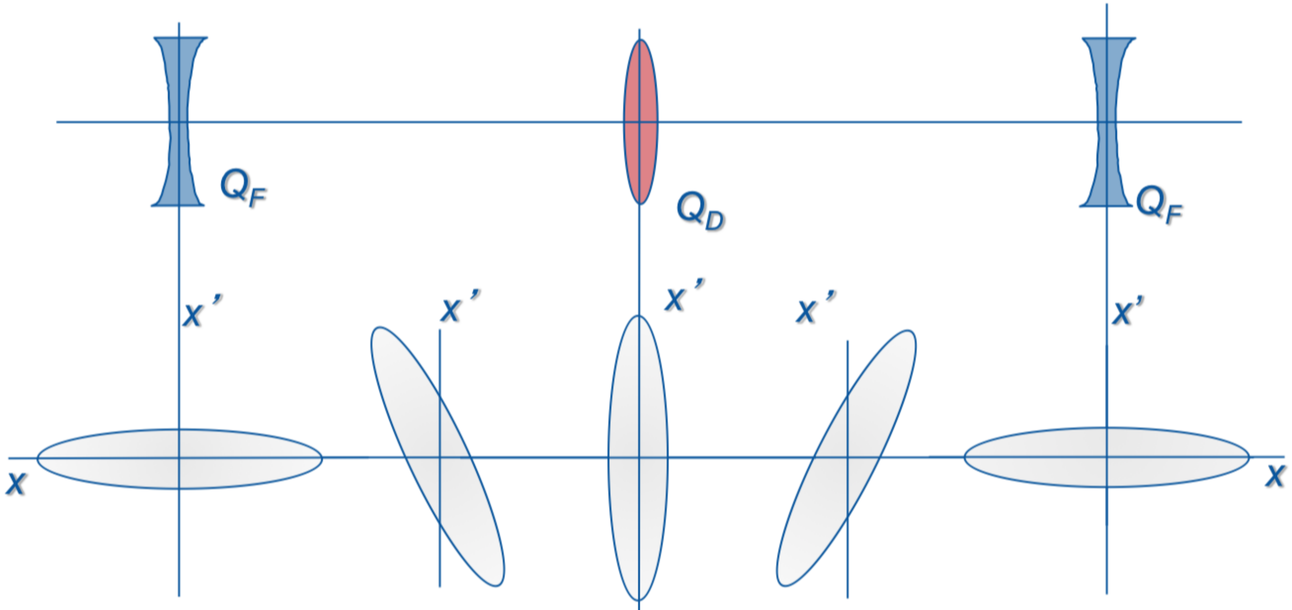


For each point along the machine the ellipse has a particular orientation, but

the area remains the same

$$A = \pi \cdot \varepsilon$$

→ *goal is a small emittance*



Different positions along the circumference of the machine

Emittance ε and β Function

The β(s) or amplitude function describes the envelope of the single-particle trajectories.

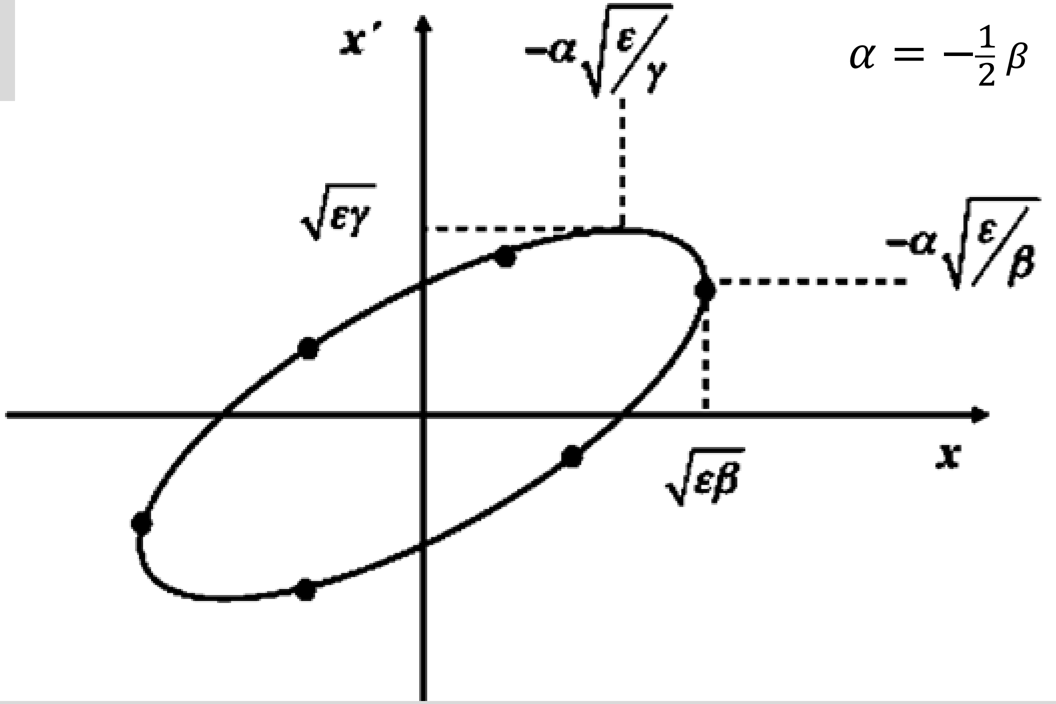
$$x(s) = \sqrt{\epsilon} \cdot \sqrt{\beta(s)} \cdot \cos(\mu(s) + \phi)$$

- s is the position along the trajectory
- $\mu(s)$ and ϕ are the amplitude in position s and ϕ its initial condition

ϵ is an invariant and describes the space occupied by the particle in the transverse two-dimensional phase space $[x, x']$.

Two important quantities that describe the beam can be introduced using the expression above:

Beam size, width:	$\sigma(s) = \sqrt{\epsilon \cdot \beta(s)}$
Beam divergence, width:	$\theta(s) = \sqrt{\epsilon / \beta(s)}$
Product (emittance):	$\sigma(s) \cdot \theta(s) = \epsilon\ (\pi!)$

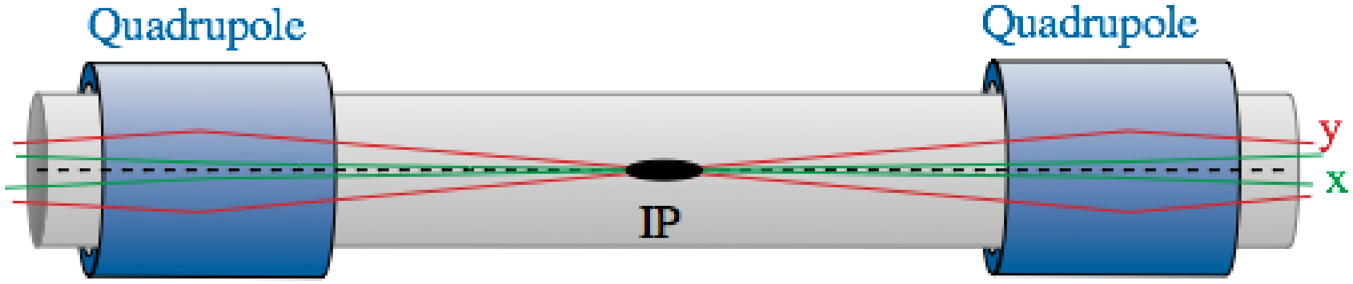


This means that emittance cannot be changed once the optics of the machine is defined: it is a property of the beam.

A narrow beam is divergent, a collimated beam is more spread

β at the Interaction Point IP

To increase the rate of collisions at the interaction point (IP), and to gain luminosity, we need beam as narrow as possible



emittance at LHC top energy is ~ = 3.75 μm rad.

$$\sigma(s) = \sqrt{\epsilon \cdot \beta(s)}$$

IP: **squeeze β to a minimum, called β*** ⇒ maximum of divergence, needs aperture → as a consequence the beam diverges → we need the beam pipe to be large enough.

Typical values at LHC are: at top E_b = 7 TeV: **ε** = 3.75 μm rad, β* = 0.55 m, **σ*** = 16.63 μm, θ*= 30 μrad

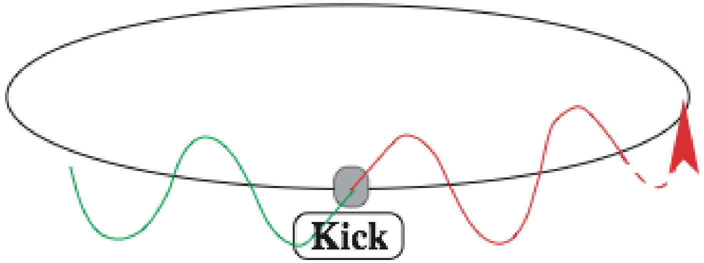
In the periodic pattern of the arc, the beta function is β = 180 m , the resulting typical beam size is therefore 0.3 mm.

- vacuum aperture of the machine pipe typically corresponds to 12 * beam size

To limit the consequences due to magnet misalignments, optics errors and operational flexibility this value is further increased to 18 times (→ about 5 centimetres).

Orbit Stability and Beam Beam Effects

Number of Oscillations in One turn: elaborate!



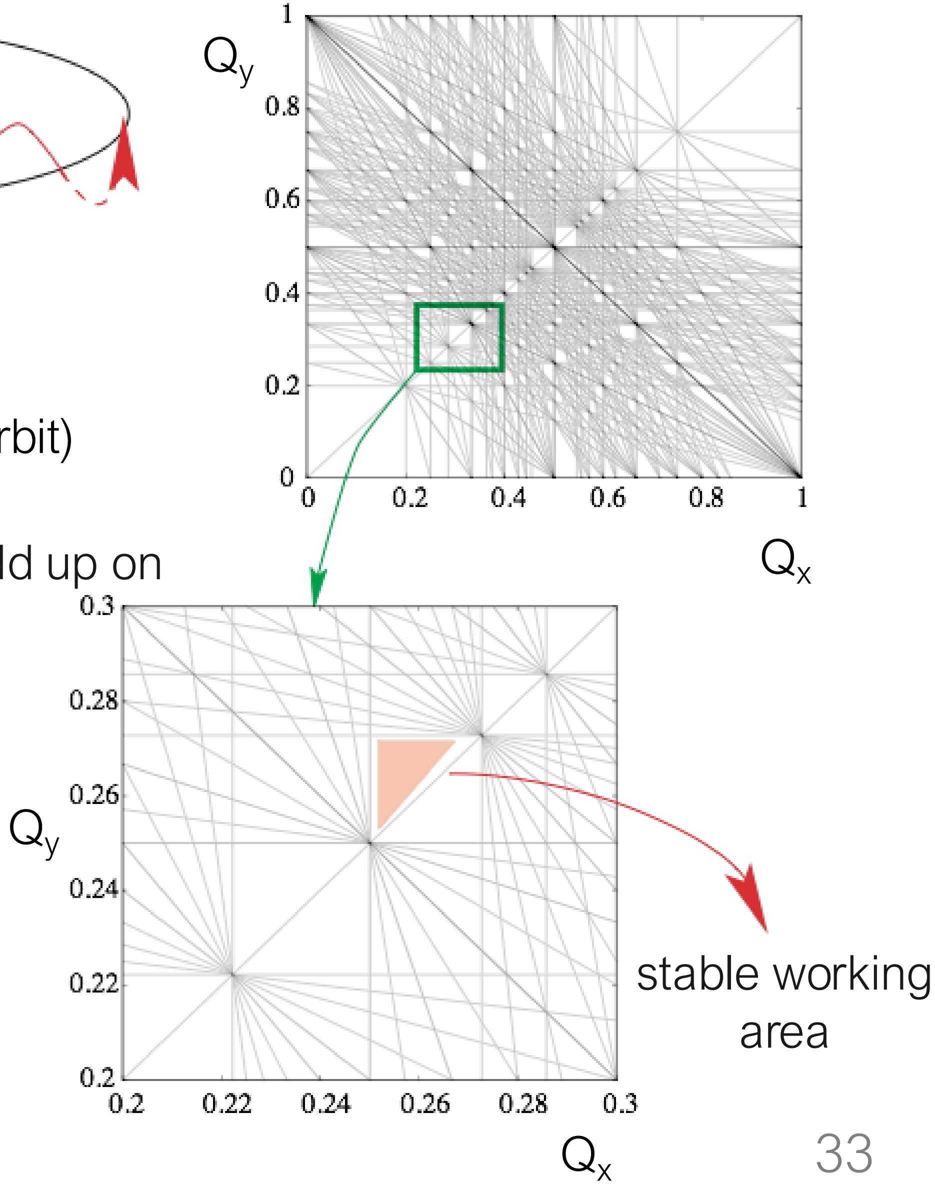
Misalignments, dipole field errors, in one point of the orbit

→ orbit perturbations

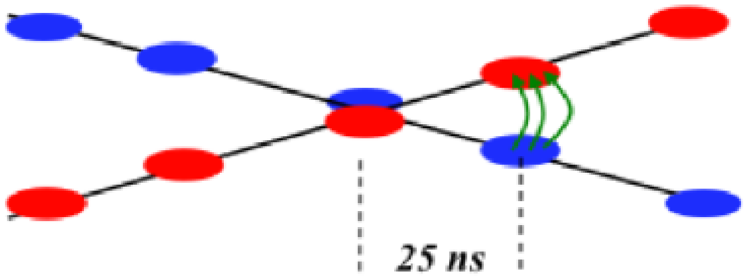
The values of the tune Q_x and Q_y (number of x and y oscillations in one orbit)

MUST not be integers

→ avoid that the beam passes always in the same 'defect'. This would add up on successive turns giving cumulative effects



The most serious limitation comes from the beam–beam interaction itself. During the collision process, individual particles of the counter-rotating bunches feel the space charge of the opposing bunch. In the case of a proton–proton collider, this strong field acts like a defocusing lens, and has a strong impact on the tune of the bunches.

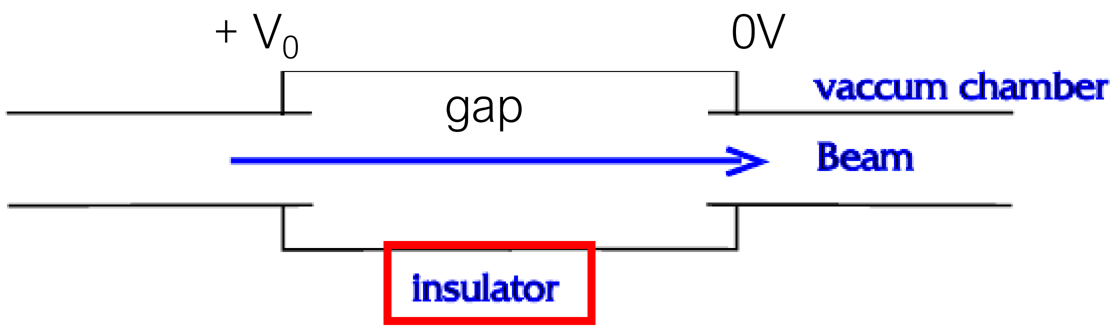


RF Systems - 0

We know how to keep particles in a closed orbit… how to accelerate them?

The Radio Frequency (RF) system is used in:

- Proton (and let's not forget...antiproton) machines
  - To accelerate/decelerate beam.
- Lepton machines:
  - To accelerate beam.
  - To compensate for energy loss due to emission of synchrotron radiation (see later).



In order to accelerate the beam of particles we need a longitudinal electric field (magnetic fields cause deflections but they do not change the overall particle momentum).
→ A longitudinal voltage, which is applied across an ***isolated*** gap in the vacuum chamber has to be used.

If we use a DC voltage, over a full turn, we get no overall acceleration, as the particle will be accelerated through the gap (+V), and decelerated over rest of the circumference (-V).

RF Systems - 1

An oscillating voltage is used,

- the particle sees an accelerating voltage in the gap
- Sees no voltage around the rest of the machine.

We must make sure that the particle always sees an accelerating voltage → the RF frequency must always be an integer multiple of the revolution frequency, which depends on the particle's *momentum*. $p \uparrow frequency \downarrow$

$$h(\text{integer}) = \text{harmonic number} = \frac{RF\ frequency}{rev.frequency} = \frac{f_{RF}}{f_{rev}}$$

Higher energy particles will have a longer orbit

- *high p → larger radius of curvature*
- *lower revolution frequency*
- *Late arrival at the accelerating cavity.*

Lower energy particles will have a shorter orbit

- *a higher revolution frequency*
- *Early arrival at the accelerating cavity.*

- For electrons we use a ~ fixed frequency as β=1 (low mass)
- Low energy protons we need a variable frequency as β<1.

frequency(β) → frequency(p)

up to when $v = c$, the velocity is = speed of light; then

frequency = constant

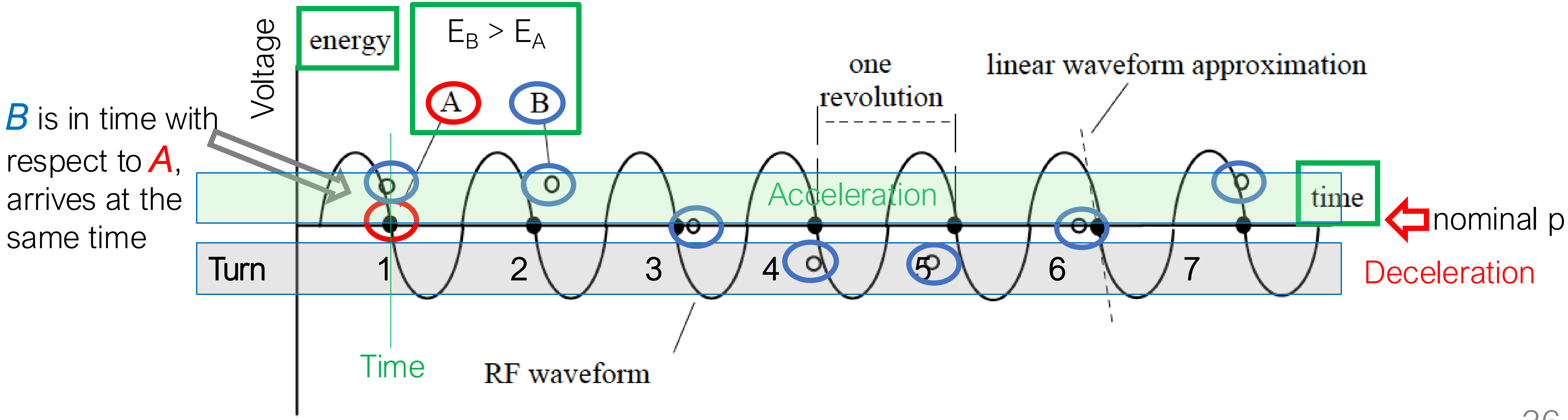
→ at 'STABLE BEAMS'

How does a particle in our machine react to this voltage? Let us start with a machine *after the acceleration phase*.

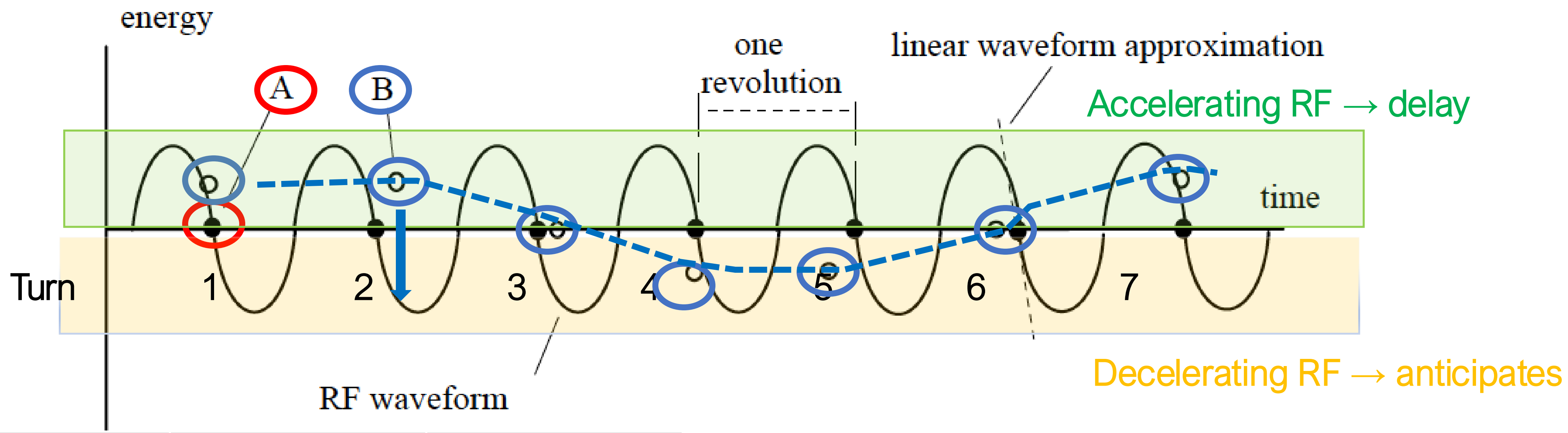
RF Systems - 2

Two particles in our accelerator:

1. particle A, has a momentum (or energy), which corresponds exactly to the RF frequency (h = 1). Suppose that when particle *A* passes through the RF cavity, the voltage is zero. In this case every time A passes through the cavity it will see zero voltage, as it's revolution frequency is the same as the RF frequency. *Particle A is synchronous with the RF voltage* **(particle is neither accelerated nor decelerated).**
2. The second particle *B*, initially, arrives at the cavity at the same time as *A*, but it's momentum is slightly higher than *A*'s, → larger bending radius → it's revolution frequency is slightly lower..



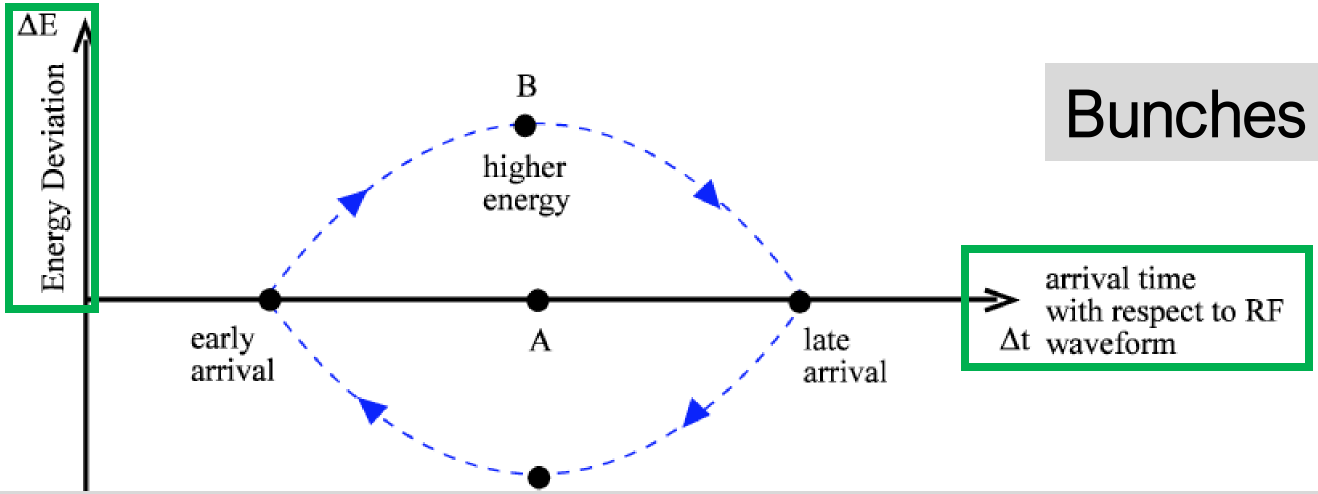
RF Systems - 3



Turn	Time wrt A	Energy wrt A
1	$t_B=t_A$	accelerated
2	$t_B>t_A$	decelerated
3	$t_B>t_A$	none
4	$t_B=t_A$	decelerated
5	$t_B<t_A$	accelerated
6	$t_B<t_A$	none
7	$t_B=t_A$	accelerated

⇦ This is the situation that we had at the beginning.

Particle A = Synchronous particle = synchronised with the RF frequency. All the other particles in the accelerator, like B, will oscillate longitudinally around A under the influence of the RF system. These oscillations are called synchrotron oscillations. This longitudinal motion is plotted in longitudinal phase space in the Figure →



Bunches formation

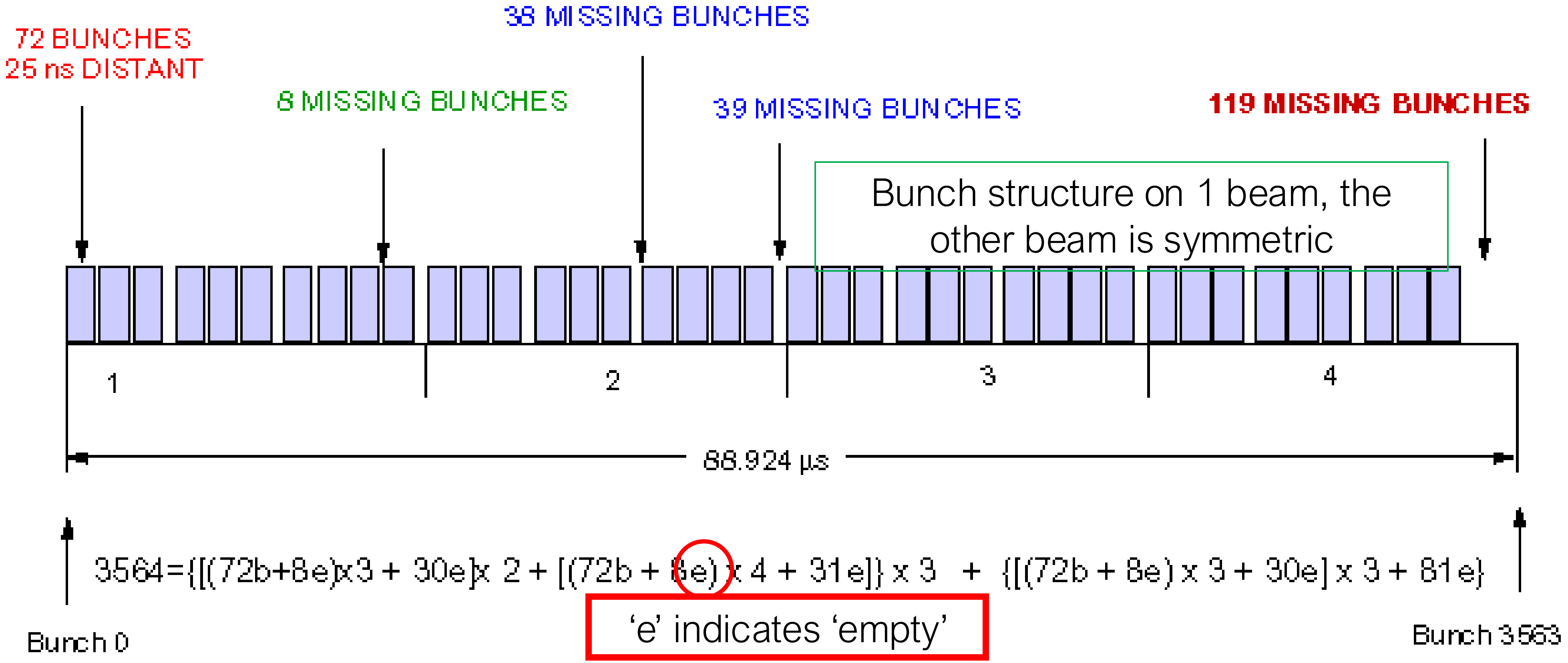
Thus *all the particles in the accelerator rotate around the synchronous particle on the longitudinal phase space plot.* → instead of being spread uniformly around the circumference of the accelerator the particles get "grouped" around the synchronous particle in a *bunch*.

This bunch is contained in an RF bucket.

- Small energy deviations → circular path inside the bunch.
- Large energy deviations → circles flatten into ellipses

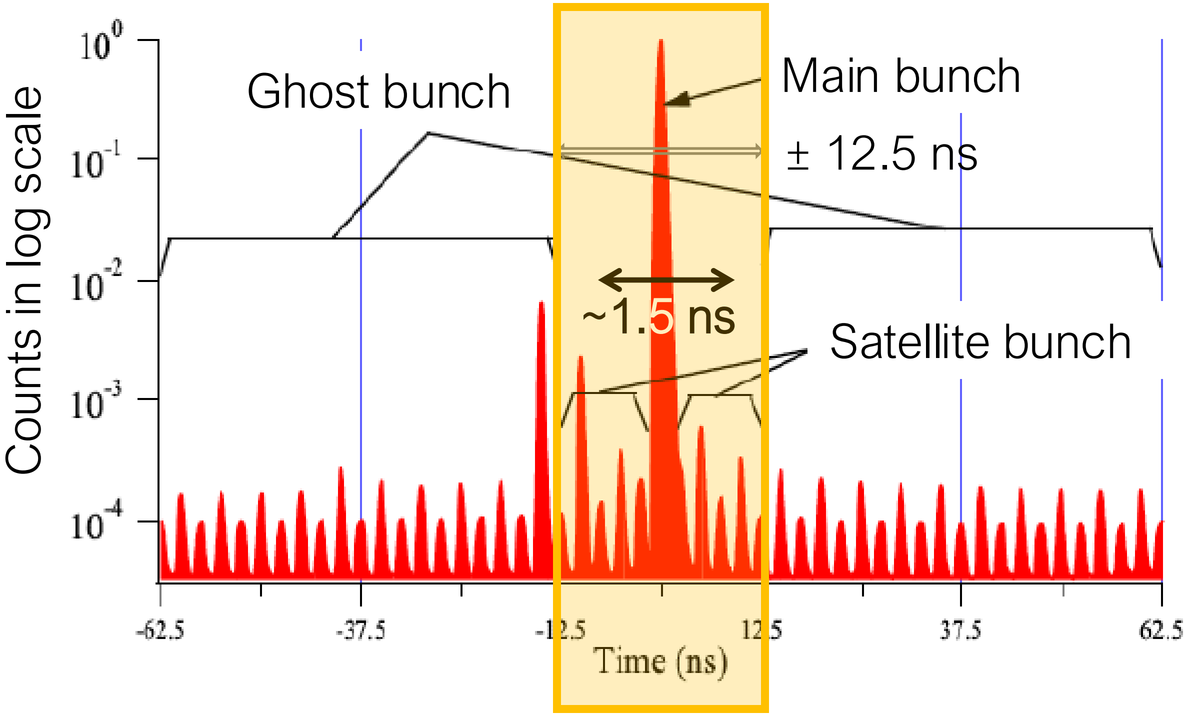
LHC Nominal Bunch Structure

A large number of bunches is injected in the machine,
(a “train”) but not all bunches collide!



Filled bunches = 2808 = { [(72b)x3]x2 + [(72b)x4] }x3+{ [(72b)x3]x3 }

Bunch Structure



- The LHC @ 400 MHz → 35640 RF bins of 2.5 ns distributed over the ring circumference.
- *One out of ten bins is filled by a bunch* → each bunch is 25 ns long (numbered from 1 to 3564).
- The slot is called Bunch Crossing ID (BCID)

The captured particles of an LHC bunch are contained within an RF bucket 1–1.5 ns long (4 sigma length). ***Main bunch***

Ideally, all particles should be contained within the nominally filled RF bins. This is correct to about 1–2% for LHC beams.
The small bunches in those RF bins which are within the 12.5 ns range around the centre of main bunch are called ***satellite bunches***, those which are outside this range are called ***ghost bunches***.

To obtain a precise measurement of the current (will see later why!) it is necessary to consider the full longitudinal distribution of the two rings.

Conflicting Arguments:

- target experiments: the extracted beam sees a VERY dense target
- in the case of two colliding beams the event rate is basically determined by the transverse particle density that can be achieved at the IP: much much lower.
- if you have beams stored into a collider machine you can bring them into collisions a very large number of times.

The luminosity $\mathcal{L}$ is a machine parameter which summarises the capacity of producing collisions:

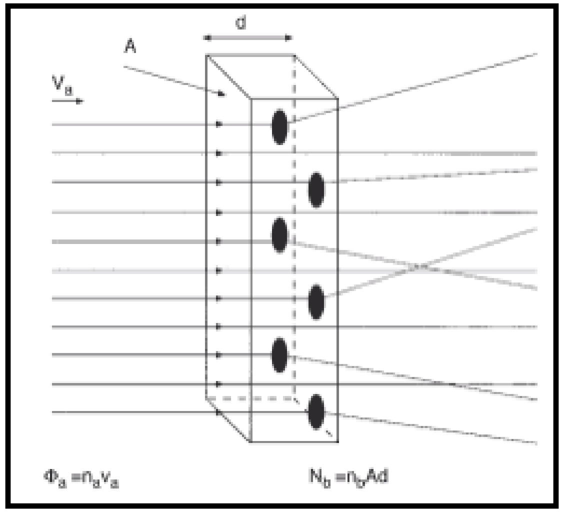
$$\dot{N} = \sigma \cdot \mathcal{L}$$

- $\dot{N}$ is the rate of events generated by the collisions of the two beams (*Number of scattering events per unit time*)
- σ is the cross-section of the process you are studying and $\mathcal{L}$ is the '*luminosity*, a machine parameter.

The total number of events of a given process with cross-section σ will be given by the integral in time of the luminosity (total integrated luminosity)

$$N = \sigma \cdot \int \mathcal{L}(t)dt$$

The Luminosity - 2



$$\Phi_a = \frac{\dot{N}_a}{A} = n_a v_a$$

Φ_a : flux
n_a : density of particle beam
v_a: velocity of beam particles

$$\dot{N} = \Phi_a \cdot N_b \cdot \sigma_b$$

N : reaction rate
N_b : target particles within beam area
σ_a : effective area of single scattering center

$$L = \Phi_a \cdot N_b$$

L : luminosity

Colliders

rate of events $\dot{N} \equiv L \cdot \sigma$ Instantaneous Luminosity

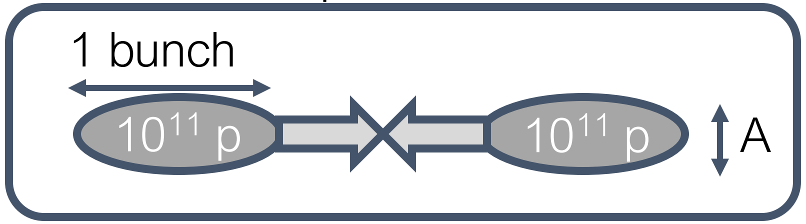
$$N = \sigma \cdot \underbrace{\int L\,dt}$$

$\sigma = N/L$ Integrated luminosity

$$\Phi_a = \frac{\dot{N}_a}{A} = \frac{N_a \cdot n \cdot v/U}{A} = \frac{N_a \cdot n \cdot f}{A}$$

$$L = f\frac{nN_aN_b}{A} = f\frac{nN_aN_b}{4\pi\sigma_x\sigma_y}$$

Collider experiment:



LHC:

$N_{a,b}$	~	10^{11}
A	~	.0005 mm^2
n	~	2800
f	~	11 kHz
L	~	10^{34} cm^{-2} s^{-1}

N_a: number of particles per bunch (beam A)
N_b: number of particles per bunch (beam B)
U: circumference of ring
n : number of bunches per beam
v : velocity of beam particles
f : revolution frequency
A: beam cross-section
σ_x : standard deviation of beam profile in x
σ_y: standard deviation of beam profile in y

The Luminosity - Continued

$$L = f\frac{nN_aN_b}{A} = f\frac{nN_aN_b}{4\pi\sigma_x\sigma_y}$$

These quantities have to be measured.
A special technique has been developed for
$\sigma_x\sigma_y$
→ Van der Meer Scan

Quantity	Status
N_a: number of particles per bunch (beam A)	Beam monitor instrumentation
N_b: number of particles per bunch (beam B)	Beam monitor instrumentation
U: circumference of ring	Exact number known
n: number of bunches per beam	Exact number known
v:velocity of beam particles	Exact number known
f:revolution frequency	Exact number known
A:beam cross-section	To be measured
σ_x :standard deviation of beam profile in x	To be measured
σ_y:standard deviation of beam profile in y	To be measured

Van der Meer Scan: Measuring the # of protons

First step for measuring the luminosity is the measurement of the current of the two beams

DCCT Beam Current Transformer

An accurate measurement of the current → number of particles in each bunch. At LHC this is done with two devices:

- DC Beam Current Transformer (DCCT) which gives a measure of the *total circulating charge* (picture to the left);
- Fast Beam Current Transformer (FBCT) which measures the *fraction of charge in each bunch*.

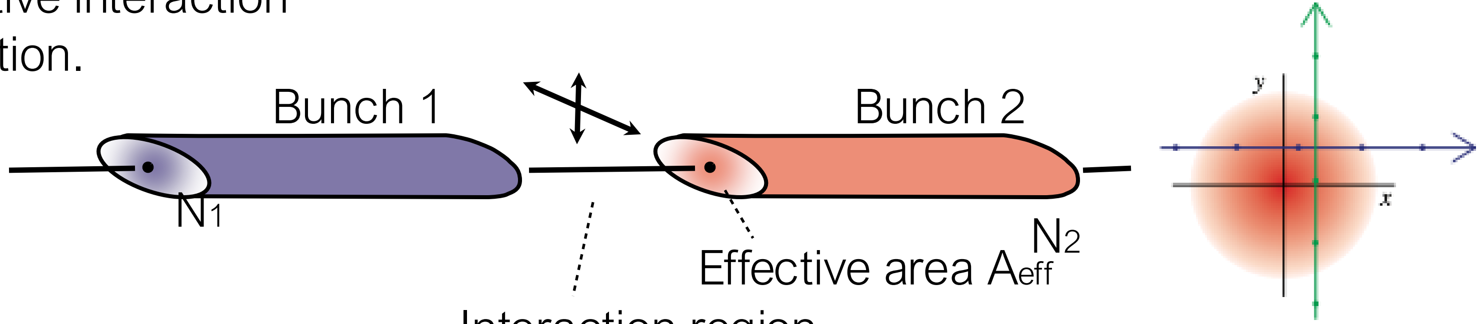
In 2010 the uncertainty on the bunch current dominated the luminosity measurement with ~10%. This reduced to below 0.5% today

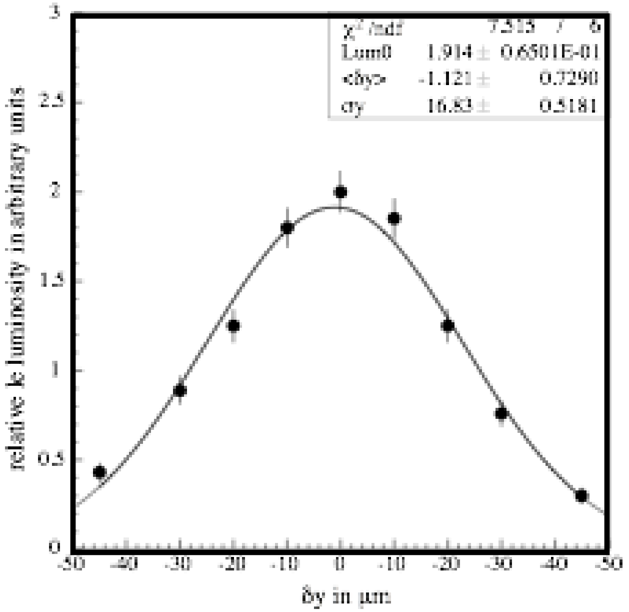
$$L = f\frac{nN_aN_b}{A} = f\frac{nN_aN_b}{4\pi\sigma_x\sigma_y}$$

Van der Meer Scan: Measuring the Size of the Beam

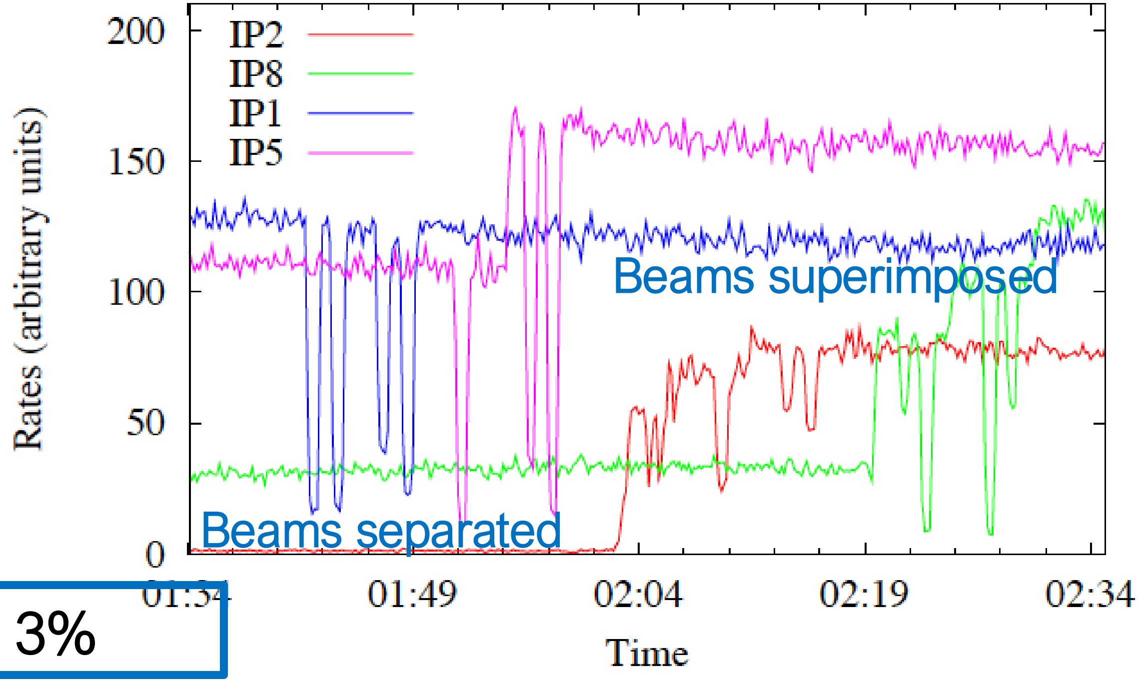
The beam size σ_x, σ_y, is determined by measuring size and shape of the interaction region → record relative interaction rates as a function of transverse beam separation.

[LHC Project Report 1019: H. Burkhardt et al.]





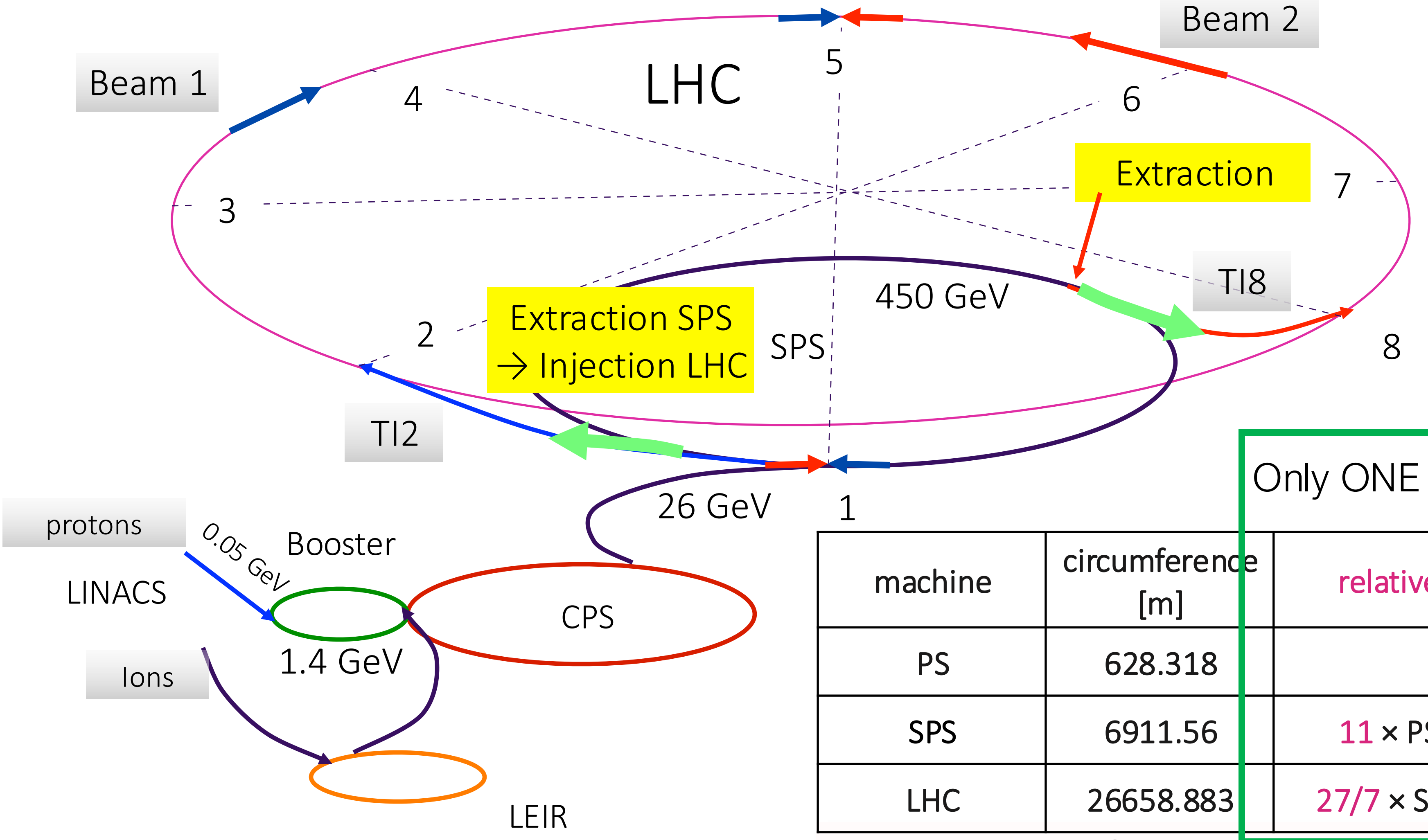
Two (~ independent) Gaussian functions in the two projections



$$\frac{L}{L_0} = \exp\left[-\left(\frac{\delta_x}{2\sigma_x}\right)^2 - \left(\frac{\delta_y}{2\sigma_y}\right)^2\right]$$

Precision on luminosity using Ven der Meer Scan ~< 3%

The CERN accelerator complex : injectors and transfer



Only ONE clock

machine	circumference [m]	relative
PS	628.318	
SPS	6911.56	11 × PS
LHC	26658.883	27/7 × SPS

simple rational fractions for synchronization based on a single frequency generator at injection

Beam size of protons decreases with energy : area $\sigma^2 \propto 1 / E$
Beam size largest at injection, using the full aperture

LHC Summary 2010-2018

2009first collisions, mostly at injection energy 2x450 GeV

2010 2x3.5 TeV, β*= 3.5 m, L_{peak} = 0.2×10^{33} cm^{-2}s^{-1} ∫ L dt = 0.044 fb^{-1} 368 bunches
2011 2×3.5 TeV, β*= 1.0 m, L_{peak} = 3.5×10^{33} cm^{-2}s^{-1} ∫ L dt = 6.1 fb^{-1} 1380 bunches
20122×4.0 TeV, β*= 0.6 m, L_{peak} = 7.7×10^{33} cm^{-2}s^{-1} ∫ L dt = 23.3 fb^{-1} 1380 bunches

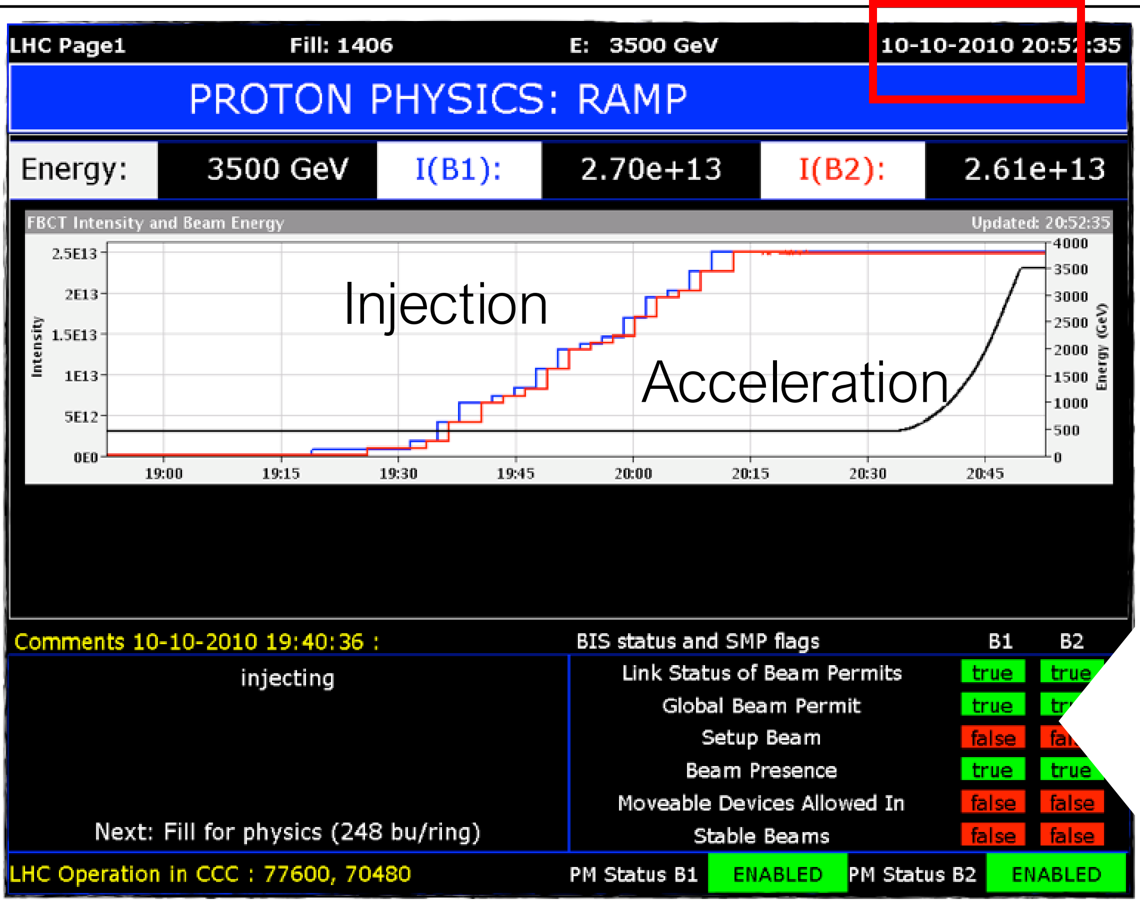
β* is ~the longitudinal size of the bunch

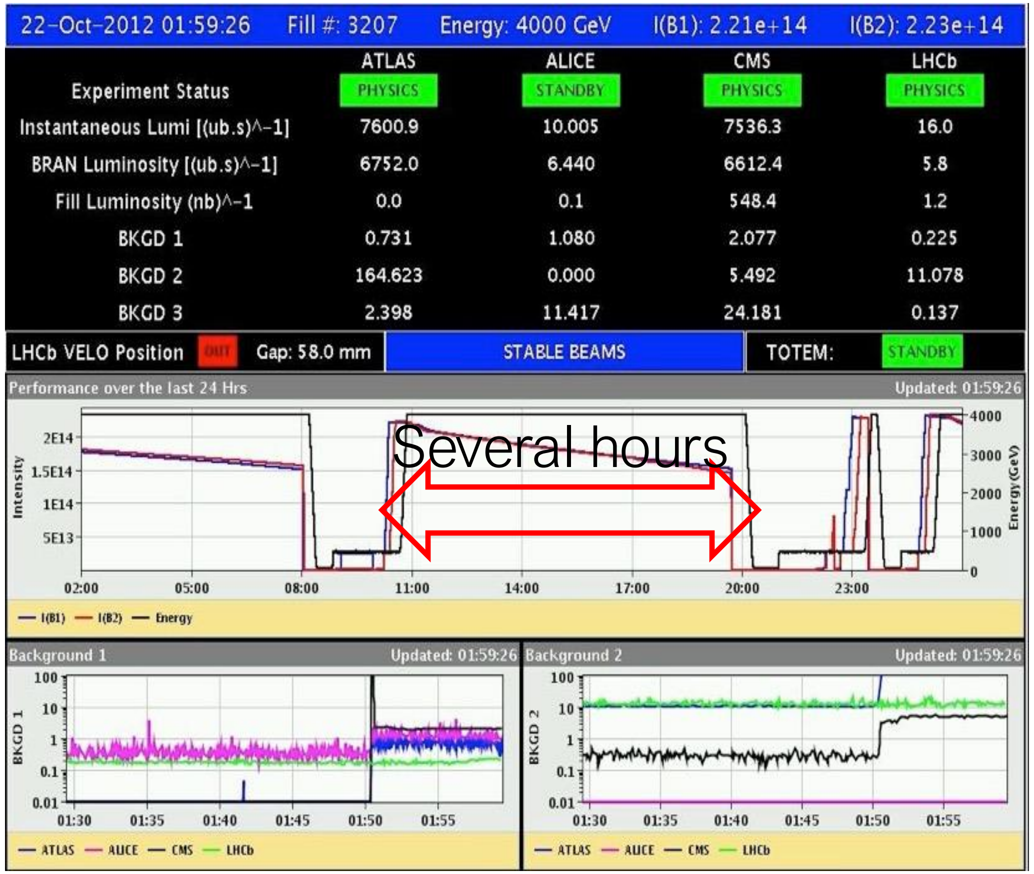
2013 2014 shutdown, magnet interconnections

2015 2×6.5 TeV, β*= 0.6 m, L_{peak} = 0.5×10^{34} cm^{-2}s^{-1} ∫ L dt = 4.2 fb^{-1} 2232 bunches
2016 2×6.5 TeV, β*= 0.4 m, L_{peak} = 1.4×10^{34} cm^{-2}s^{-1} ∫ L dt = 35.6 fb^{-1} 2208 bunches
2017 2×6.5 TeV, β*= 0.3 m, L_{peak} = 2.1×10^{34} cm^{-2}s^{-1} ∫ L dt = 50.4 fb^{-1} 2544 bunches
2018 2×6.5 TeV, β*= 0.3 m, ∫ L dt = 60 fb^{-1} 2556 bunches

	LHC design	achieved
Momentum at collision, TeV/c	7	6.5
Luminosity, cm^{-2}s^{-1}	1.0E+34	2.4 E+34
Dipole field at top energy, T	8.33	8.33
Number of bunches, each beam	3564	2556
Particles / bunch	1.15E+11	1.7E+11
Typical beam size in ring, µm	200 – 300	~300
Beam size at IP, µm	17	16

LHC in Operation: what you see in Control Room

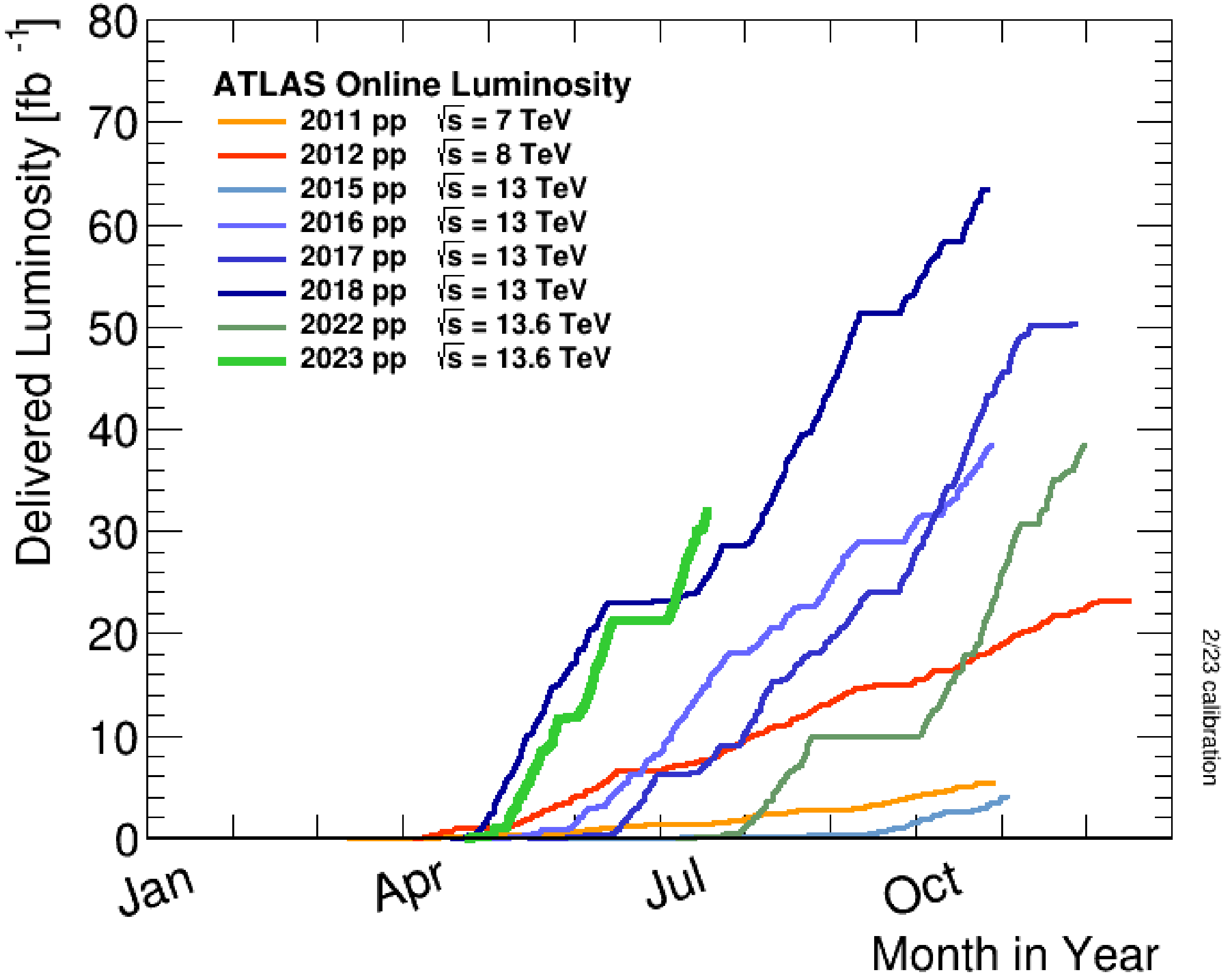


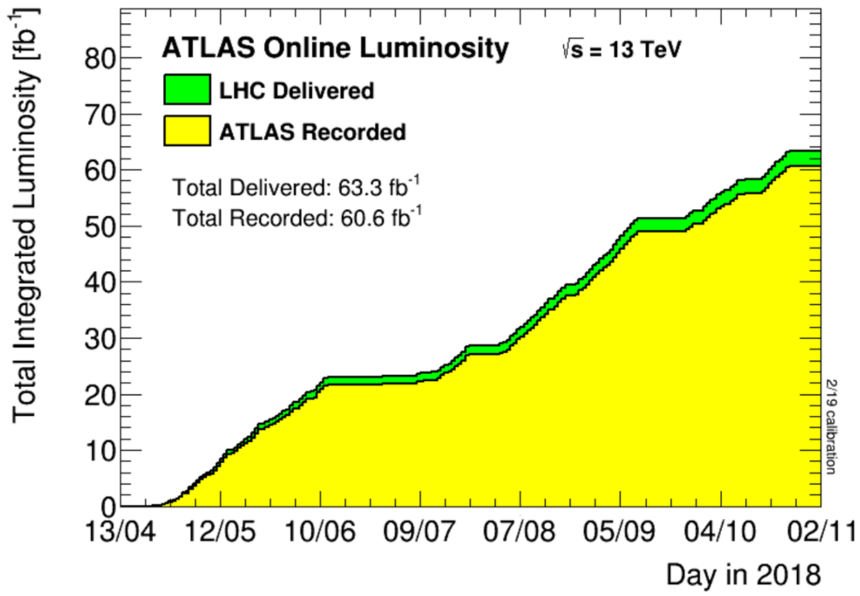


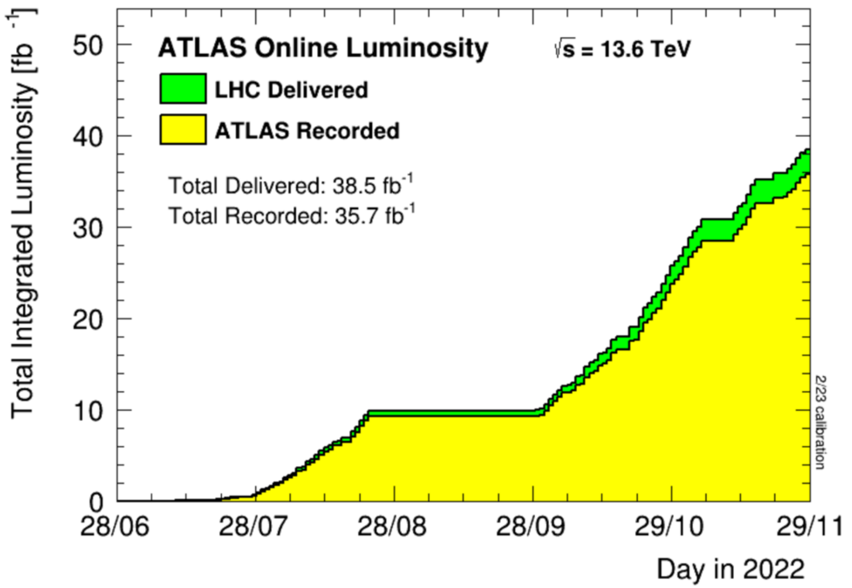
- Injection
- Ramp
- Squeeze
- Adjust
- Stable Beams

Fill

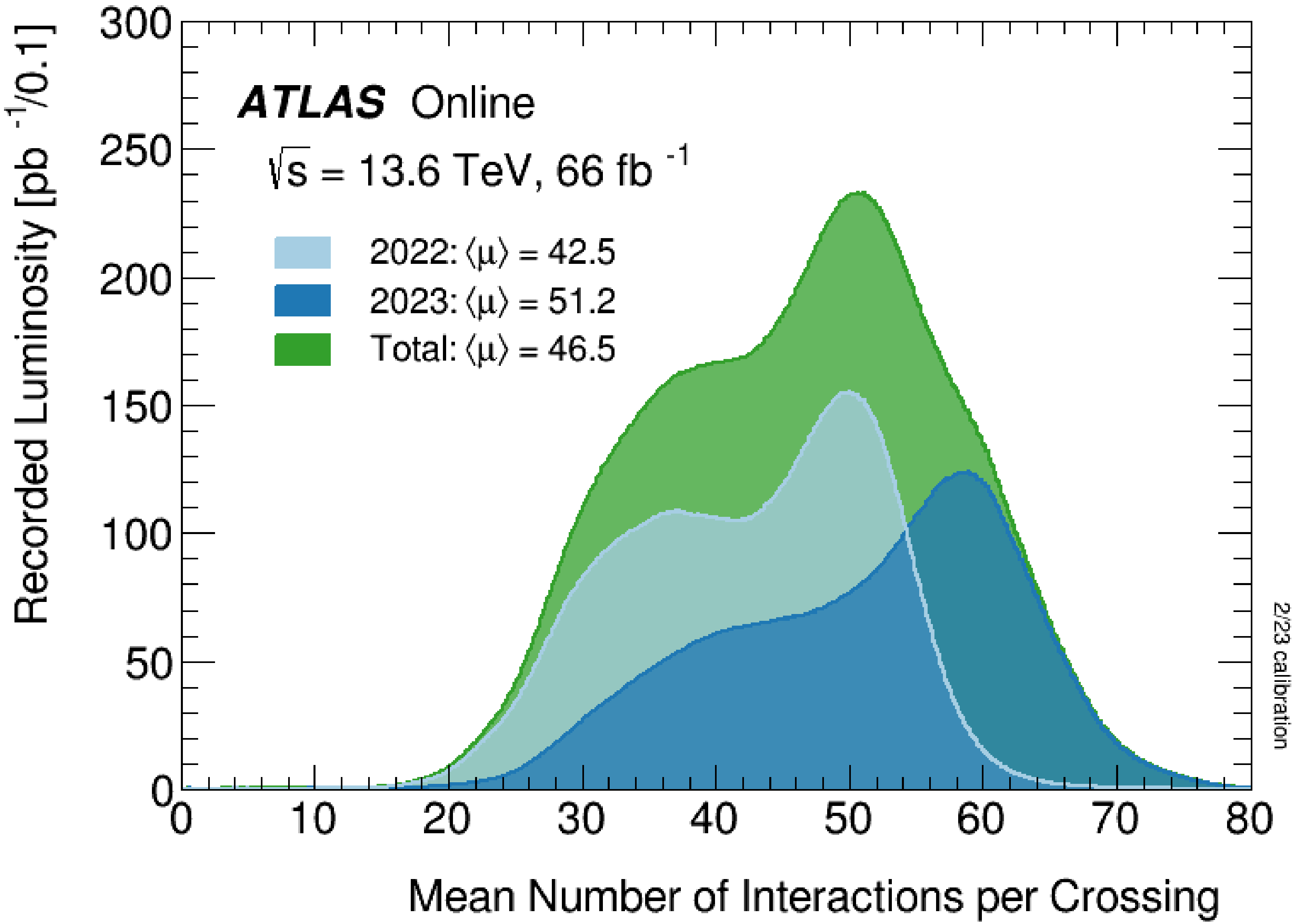
2016→2018 (Run2),2022,2023 (Run3) LHC Luminosity







Pile-up

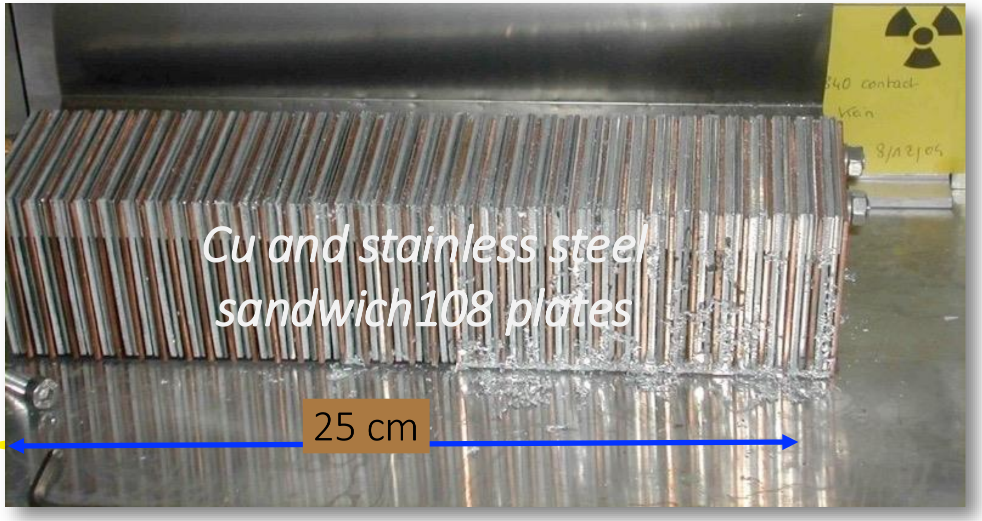


ATLAS Online
√s = 13.6 TeV, 66 fb -1
2022: ⟨μ⟩ = 42.5
2023: ⟨μ⟩ = 51.2
Total: ⟨μ⟩ = 46.5
Recorded Luminosity [pb -1/0.1]
Mean Number of Interactions per Crossing
2/23 calibration
0
50
100
150
200
250
300
0
10
20
30
40
50
60
70
80

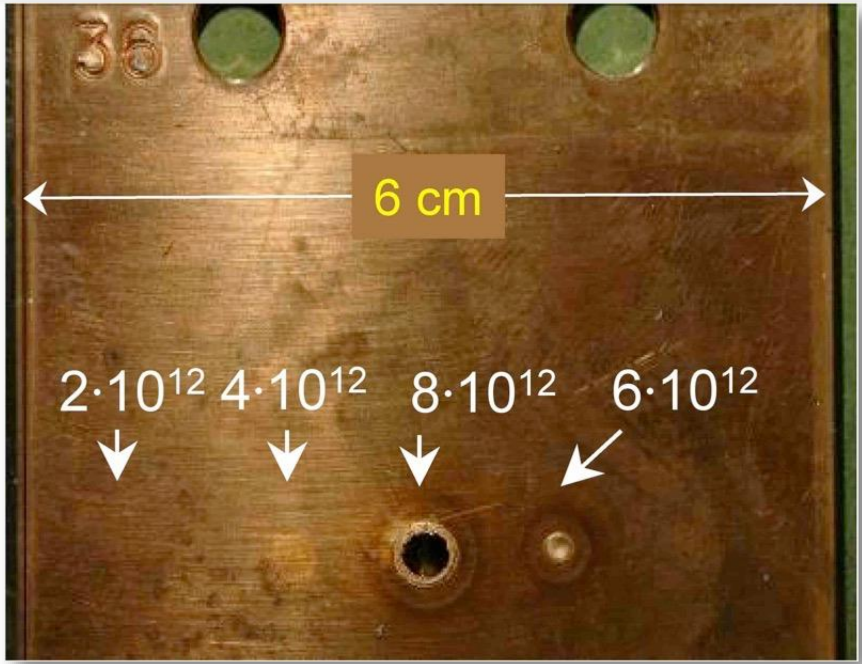
Damage potential : SPS experiment

Controlled experiment with beam extracted from SPS at 450 GeV in a single turn, with perpendicular impact on Cu + stainless steel target

450 GeV protons →



r.m.s. beam sizes $\sigma_{x/y} \approx 1$ mm



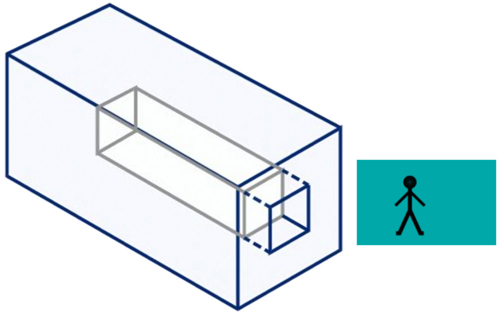
SPS results confirmed :

8×10^{12} clear damage

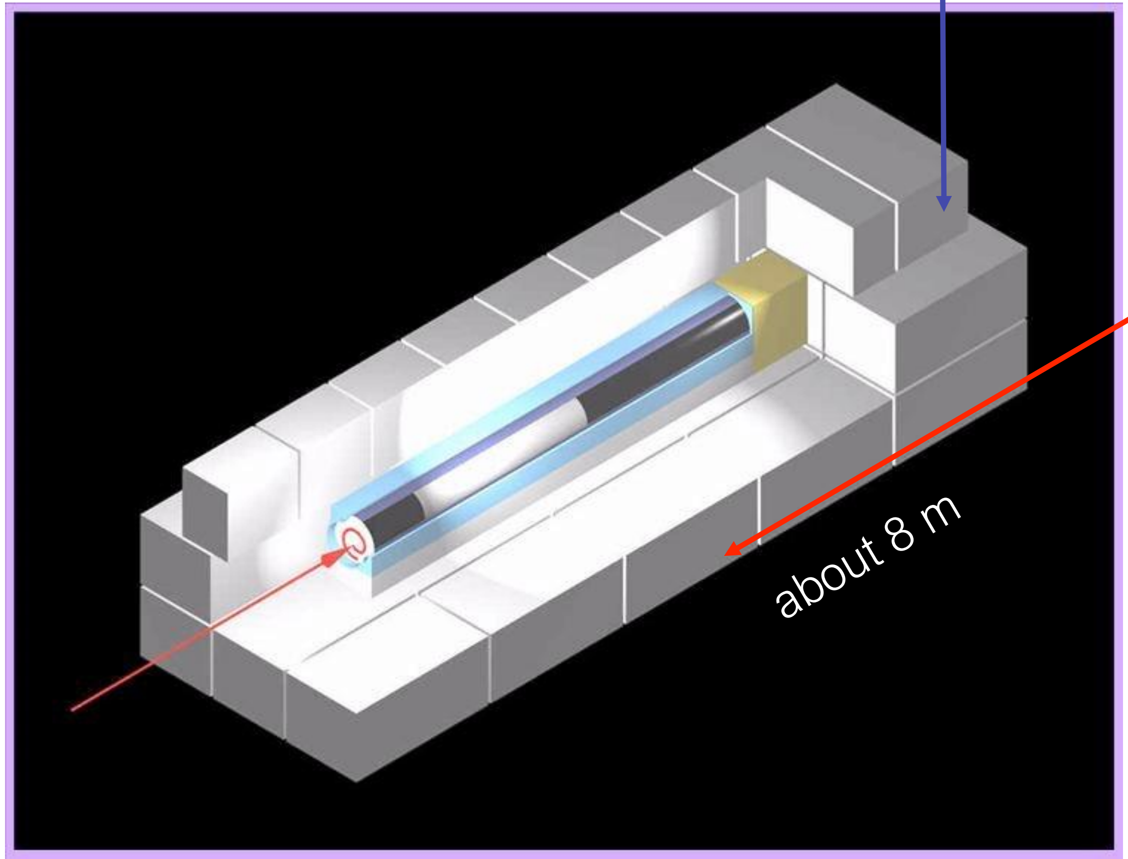
2×10^{12} below damage limit

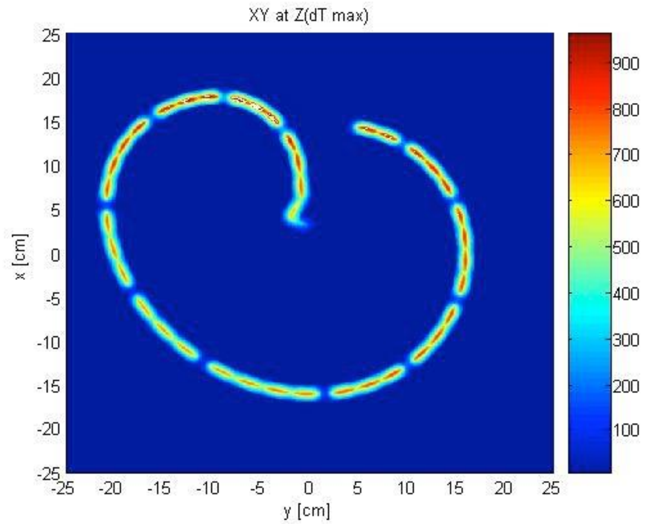
For comparison, the LHC nominal at 7 TeV : 2808 × $1.15\times10^{11} = 3.2\times10^{14}$ p/beam at $< \sigma_{x/y} > \approx 0.2$ mm over 3 orders of magnitude above damage level for perpendicular impact

Dumping the LHC Beam

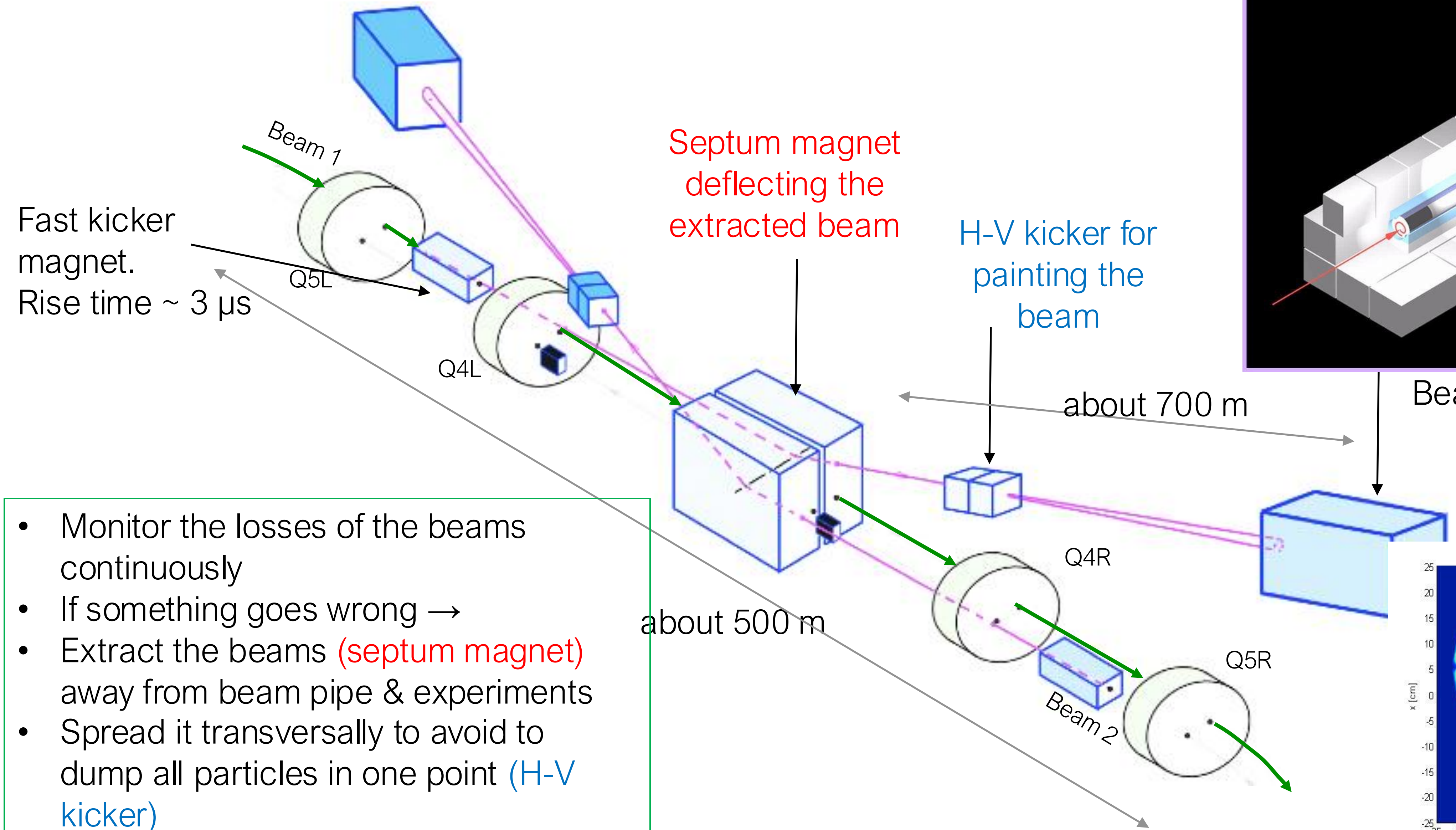


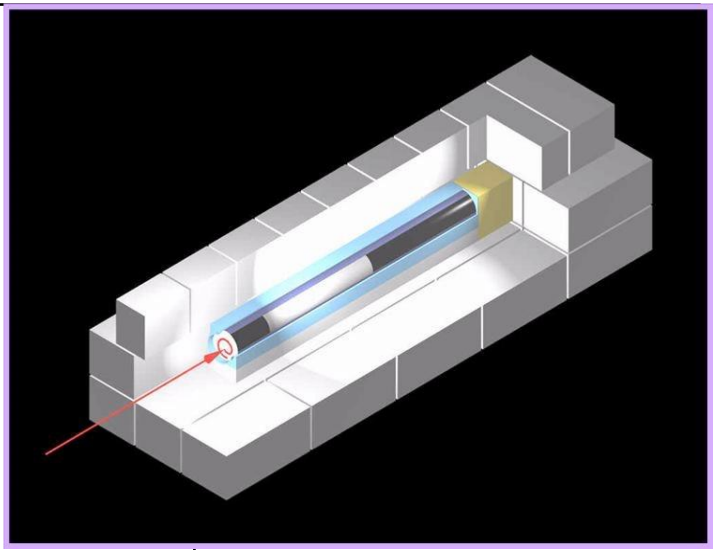
beam absorber
(graphite)

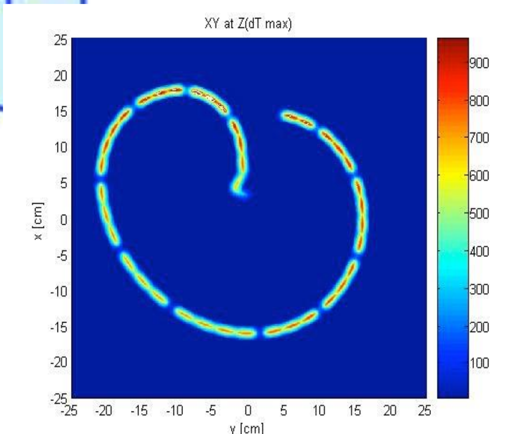




Schematic layout of beam dump system in IR6

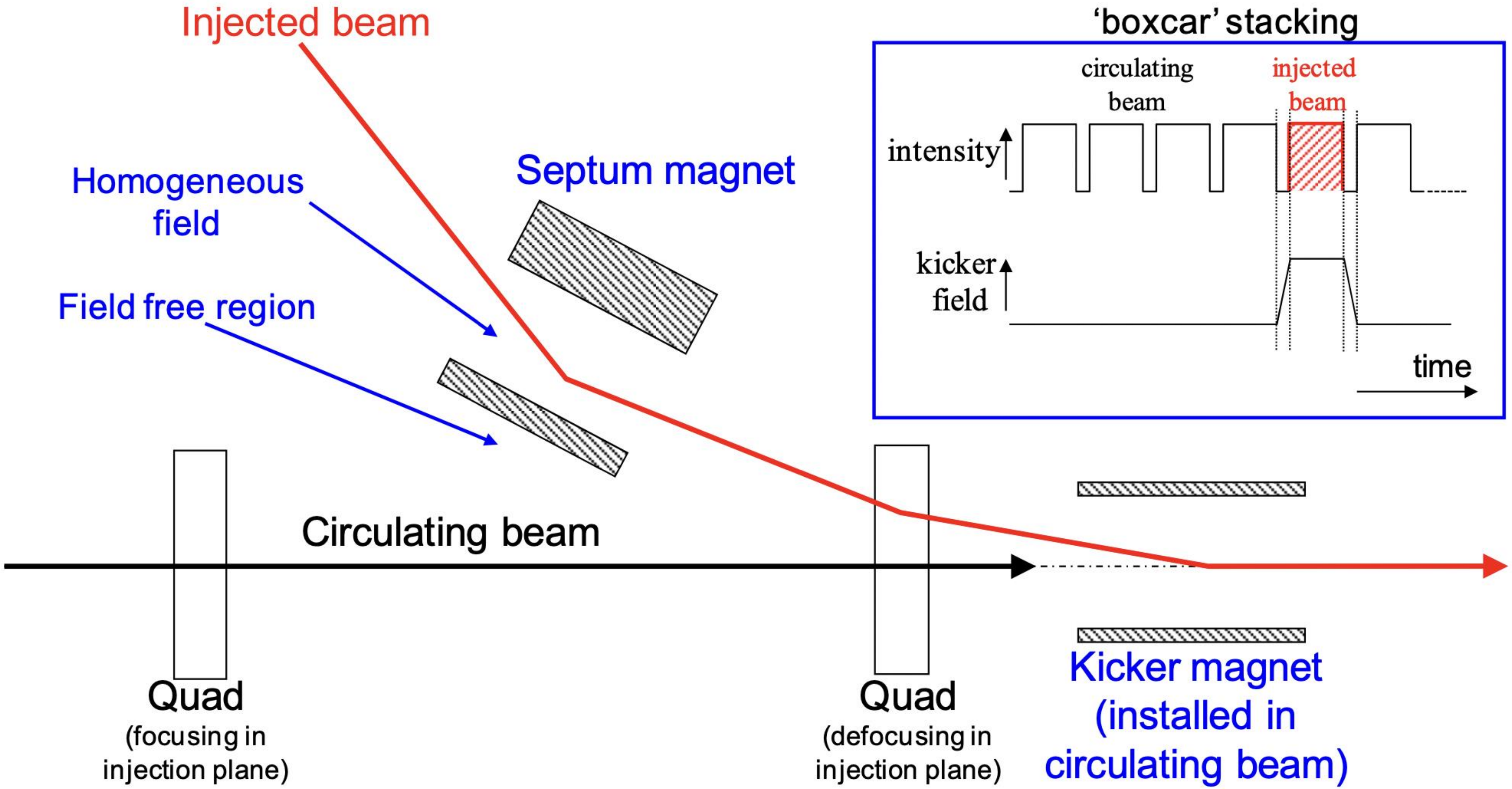




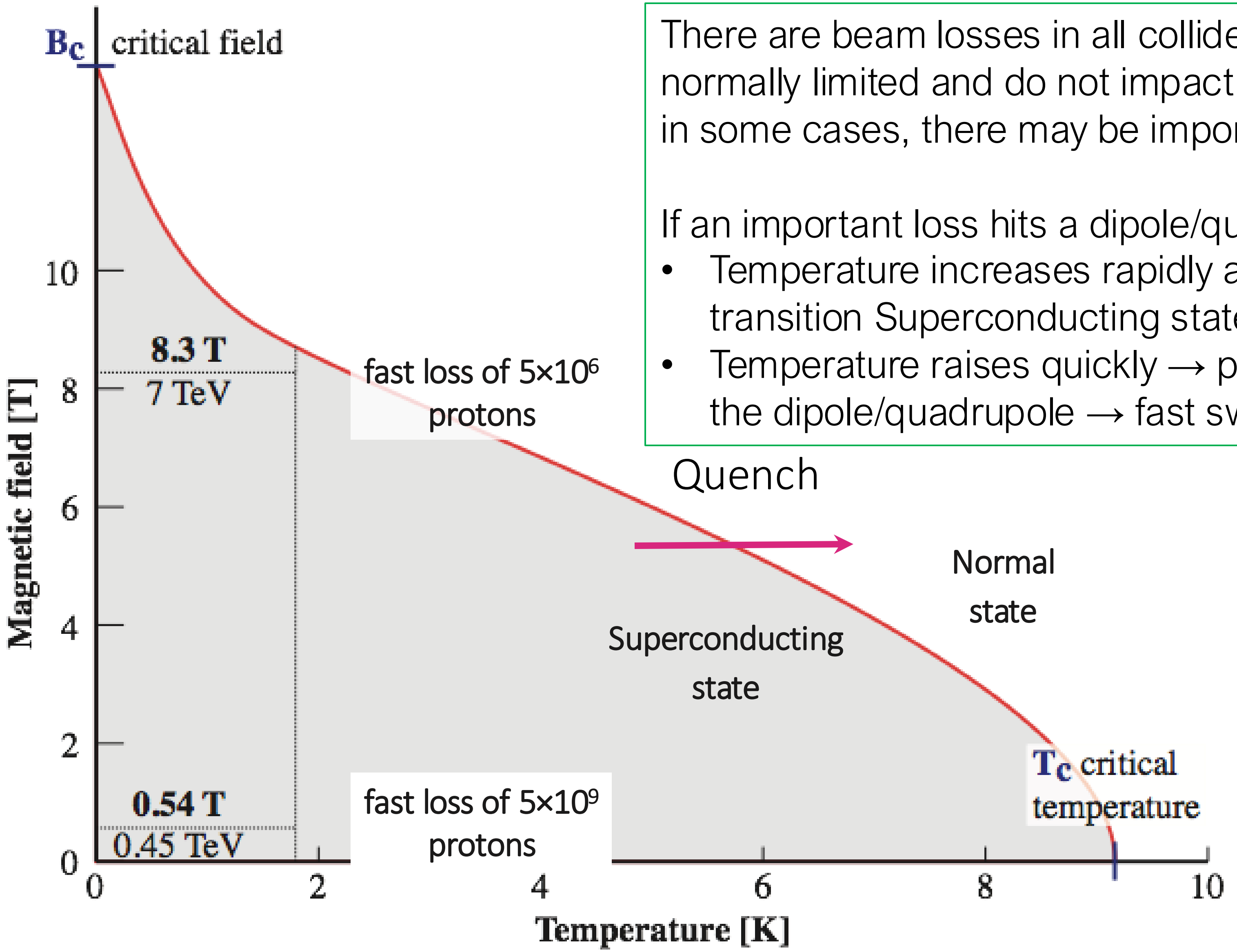


- Monitor the losses of the beams continuously
- If something goes wrong →
- Extract the beams (septum magnet) away from beam pipe & experiments
- Spread it transversally to avoid to dump all particles in one point (H-V kicker)

Septum Magnet



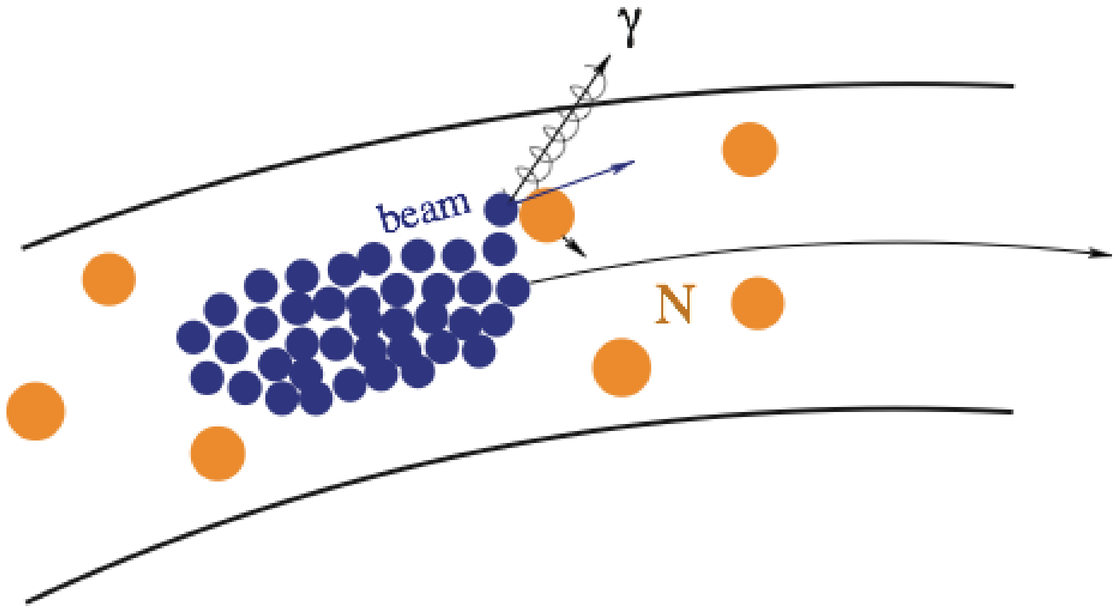
Operational Margin of a Superconductiong Dipole



There are beam losses in all colliders. These are normally limited and do not impact the accelerator but, in some cases, there may be important losses.

If an important loss hits a dipole/quadrupole →

- Temperature increases rapidly and may induce the transition Superconducting state → Normal state
- Temperature raises quickly → potential damage of the dipole/quadrupole → fast switch off



Beam blow up, core + halo
Background to experiments
loss, radiation, beam and
Luminosity lifetime

Minimize effect : Good vacuum
O(nTorr or 10^{-9} mb) Collimation

$$p = 1\,\text{ntorr} = 1.33 \times 10^{-7}\,\text{Pa}$$

$$\rho_m = \frac{p}{kT} = 3.26 \times 10^{13}\text{molecules / m}^3$$

$$\text{typical cross section } \sigma = 6\,\text{barn} = 6 \times 10^{-28}\text{m}^2$$

$$\text{collision probability } P_{\text{coll}} = \sigma\,\rho_m = 1.96 \times 10^{-14}/\,\text{m}$$

$$\tau = \frac{1}{P_{\text{coll}}\,c} = 1.7 \times 10^5\,\text{s} = 47\,\text{hours} \qquad \text{for } v \approx c$$

$$\frac{1}{\tau} = -\frac{1}{n}\frac{dn}{dt}$$

beam lifetime τ general expression
average time between collisions leading to
beam loss inverse normalized loss rate

Electron Positron Colliders

We know how to design and build hadronic storage rings (what we we discussed before)

Situation VERY different for light particles like electrons!

Synchrotron radiation (also known as magneto-bremsstrahlung radiation) is the electromagnetic radiation emitted when relativistic charged particles are subject to an acceleration perpendicular to their velocity (a ⊥ v). It is produced artificially in some types of particle accelerators or naturally by fast electrons moving through magnetic fields.

Bent on a circular path, electrons in particular radiate an intense light, the so-called synchrotron radiation
→ strong influence on the beam parameters.
The power loss due to synchrotron radiation, P_s, depends on the bending radius, on the particle mass **(4th power)** and the energy of the particle beam:

$$P_s = \frac{2}{3} \cdot \alpha \cdot \hbar \cdot c^2 \cdot \frac{\gamma^4}{\rho^2} \quad where \quad \gamma = \frac{E}{m \cdot c^2}$$

$$\rightarrow \quad \frac{P_s^e}{P_s^p} = \frac{m_e^4}{m_p^4} \approx 2000^4 \approx 2 \cdot 10^7$$

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α =1/137, ρ radius of the ring.

As a consequence, the particles will lose energy at every turn.

This expression shows why synchrotron radiation has ~ no importance for protons while it is crucial for electrons.

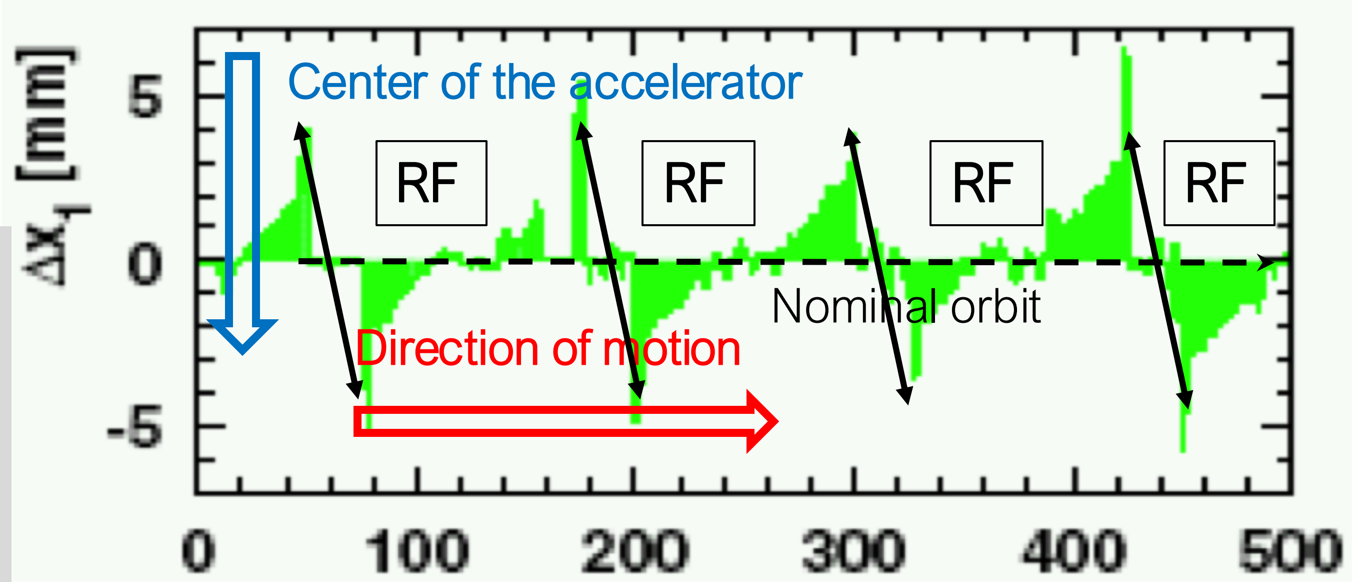
(Important observation (to be discussed later!): why not using muons? They are leptons like electrons but m_μ ~ 200 m_e → almost as good as protons for what concerns synchrotron radiation

To compensate for these losses, RF power has to be supplied to the beam at any moment.

Electron Positron Colliders: the LEP case

Large Electron–Positron Collider (LEP) storage ring: deviation from the ideal orbit towards the inside of the ring due to synchrotron radiation.

$$\frac{p}{e} = B \cdot \rho \rightarrow when\ p \downarrow also\ \rho \downarrow$$

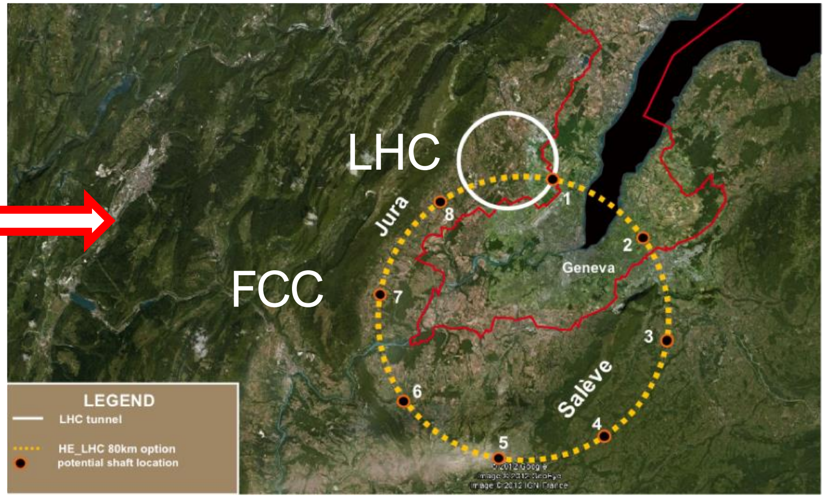


The effect on the orbit is large: up to 5 mm. To compensate for these losses, 4 RF stations were placed in the straight sections to supply the power lost in synchrotron radiation

$$P_s = \frac{2}{3} \cdot \alpha \cdot \hbar \cdot c^2 \cdot \frac{\gamma^4}{\rho^2} \quad where \quad \gamma = \frac{E}{m_e \cdot c^2}$$

Synchrotron radiation losses limit the beam energy that can be carried in a storage ring of a given size. Two options:

- Larger radii ($P_s^e \propto E^4/\rho^2$), Future Circular Collider (FCC) foresees a 100 km ring to bring electrons (and positrons) of up to 175 GeV energy where one has an energy loss of 8.6 GeV / turn, or an overall power of 47 MW of the radiated light at full beam intensity
- Linear accelerators



The BIG QUESTION: Protons vs Electrons

	Protons	Electrons
Advantages	• Energy potential very large, only limited by B-field technology and ring radius (→ cost of civil engineering) • → Large kinematic window → *Discovery machine*	• Point-like particles, no structure, defined initial quantum numbers • Energy and momentum conservations (4 equations → missing energy (say neutrinos) • *Large precision potential* (loop corrections)
Disadvantages	• Very complex object, results difficult to interpret. The energy of the proton components that scatter are significantly lower that of the beam protons • Momentum conservation only in the transverse plane • Limited precision potential	• Energy potential limited by sync.radiation → RF power • Limited discovery potential

Hadron machines → discovery (top, W, Z^0, Higgs…) later studied with high precision at $e^{\pm}$ colliders (W, Z^0…) with new measurements (number of families from the Z-line shape)

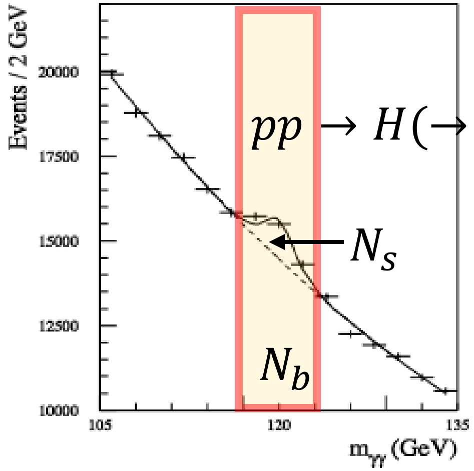
Future Upgrade(s) of LHC

There are two upgrades of the LHC machine:

1. HL-LHC approved, (High Luminosity) →increase x 10 in integrated luminosity through the use of Nb3Sn superconducting magnets, expected to start in 2026. Why high luminosity helps? See below!
2. HE-LHC being considered (High Energy). If approved it might go into operation not earlier than ~2040

	HERA (DESY)	TEVATRON* (Fermilab)	RHIC (Brookhaven)	LHC (CERN)		
Physics start date	1992	1987	2001	2009	2015	2026 (HL-LHC)
Physics end date	2007	2011	—	—		
Particles collided	ep	$p\bar{p}$	pp (polarized)	pp		
Maximum beam energy (TeV)	e: 0.030 p: 0.92	0.980	0.255 55% polarization	4.0	6.5	7.0
Maximum delivered integrated luminosity per exp. (fb^{-1})	0.8	12	0.38 at 100 GeV 1.3 at 250/255 GeV	23.3 at 4.0 TeV 6.1 at 3.5 TeV	94.5	250/y
Luminosity (10^{30} $cm^{-2}s^{-1}$)	75	431	245 (pk) 160 (avg)	7.7×10^3	2×10^4	5.0×10^4 (leveled)

	HE-LHC
	pp
(TeV)	13.5
(km)	26.7
gions	2 (4)
egrated luminosity $^{-1}$/year)	1.0
Peak luminosity (10^{34} cm^{-2} s^{-1})	28



An increase in luminosity gives a larger rate of events:
the 'statistical significance' of a 'N_s signal events' (the channel you are interested in) over 'N_b background events' (~similar to signal) is given by

$$S = \frac{N_s}{\sqrt{N_b}} \propto \sqrt{\Delta \mathcal{L}}$$

This allows to better establish known channels or to find new ones

Future Accelerators

$e^{\pm}$ colliders	Where	Beam Energy(TeV)	Circ.(Km) Length(Km)	h colliders	Where	Energy(TeV)	Circ.(Km)
FCC-ee	CERN	.46,.12,.183	100	FCC-pp	CERN	50	100
CEPC	China	.46,.120	100	SPPC	China	37.5	100
ILC	Japan	.125,.250	20.5/21	HE-LHC	CERN	13.5	26.7
CLIC	CERN	0.19,1.5	60				

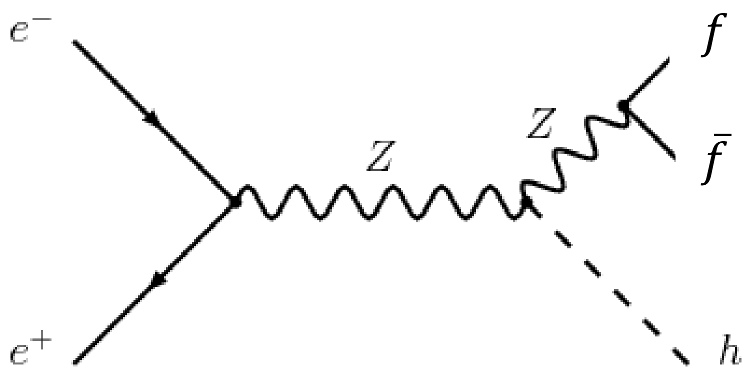
	FCC-ee	CEPC	ILC	CLIC
Species	e^+e^-	e^+e^-	e^+e^-	e^+e^-
Beam energy (GeV)	46, 120, 183	46, 120	125, 250	190, 1500
Circumference / Length (km)	97.75	100	20.5, 31	11, 50

	LHeC	HE-LHeC	HE-LHC	FCC-hh	SPPC	μ collider
Species	ep	ep	pp	pp	pp	$\mu^+\mu^-$
Beam Energy (TeV)	0.06(e), 7 (p)	0.06(e), 13.5 (p)	13.5	50	37.5	0.063, 3
Circumference (km)	9(e), 26.7 (p)	9(e), 26.7 (p)	26.7	97.75	100	0.3, 6
Interaction regions	1	1	2 (4)	4	2	1, 2
Estimated integrated luminosity per exp. (ab^{-1}/year)	0.1	0.1	1.0	0.2–1.0	0.4	0.001, 1.0
Peak luminosity (10^{34} cm^{-2} s^{-1})	0.8	1.2	28	5–30	10	2.2, 71

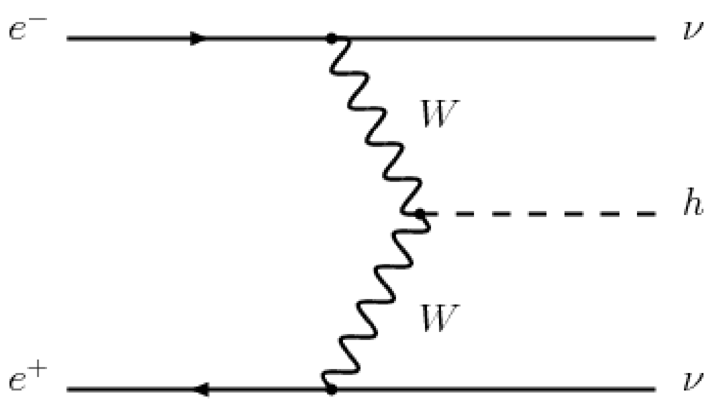
Higgs Production at LEP (e^+e^- Collider)

Production of Higgses at LEP:

- The *Higgsstrahlung* mechanism
- The *WW fusion* diagram (& ZZ fusion mechanism)



Higgsstrahlung



WW fusion

$\sigma_{Higgsstrahalung}$ >>> $\sigma_{WW\ fusion}$

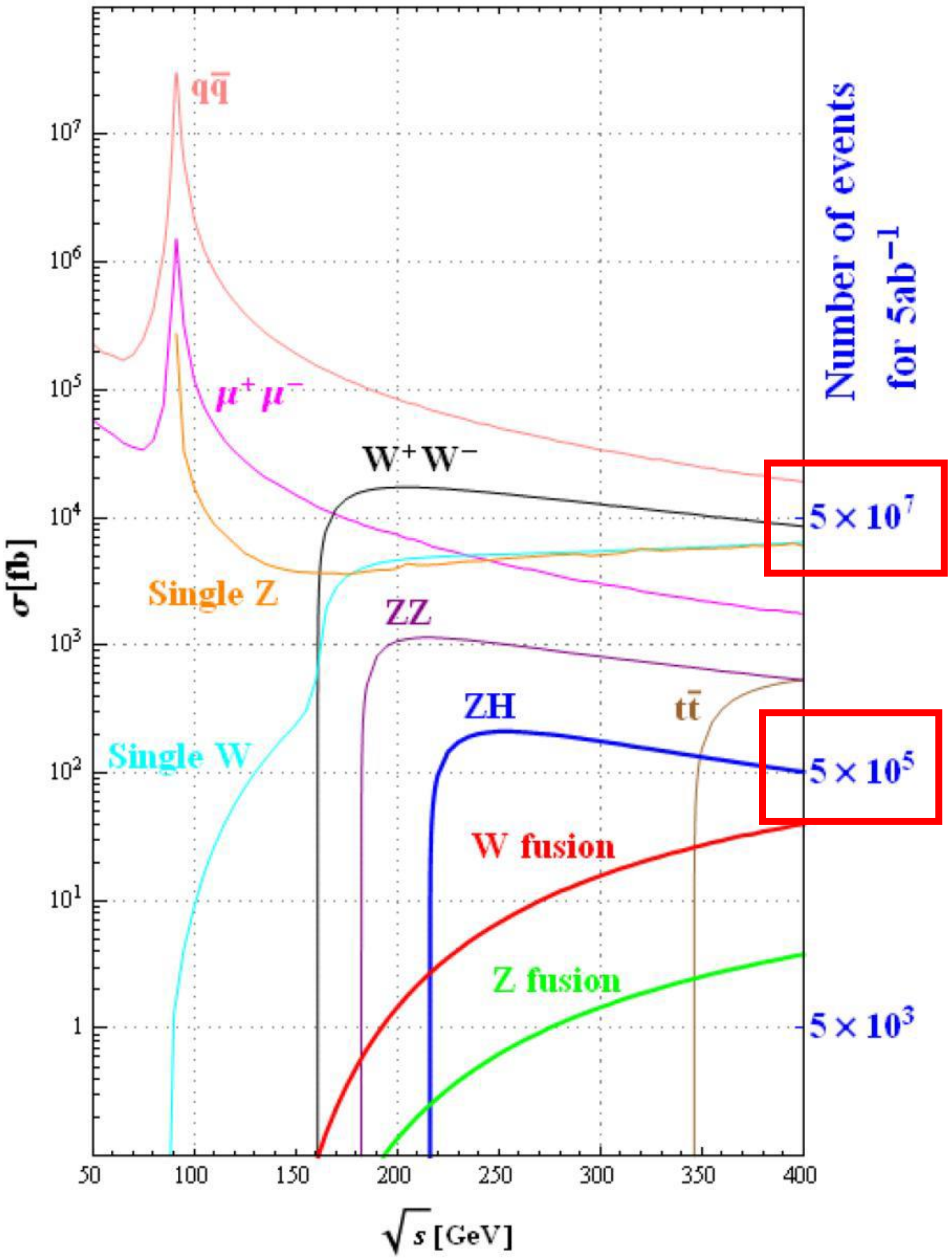
kinematic limit: cms energy used to produce m_Z and $m_H \rightarrow m_H^{max} = \sqrt{s} - m_Z$ (…some margin by the tail of the Breit-Wigner distribution)

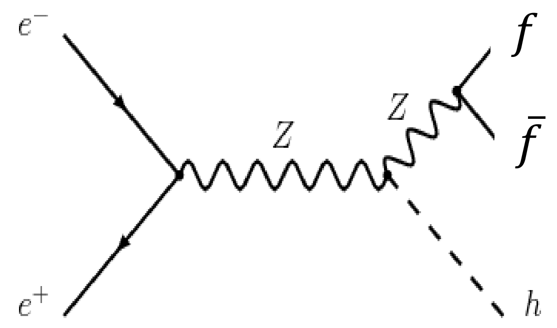
can produce H up to $\sqrt{s}$ however small cross section limits drastically the statistics

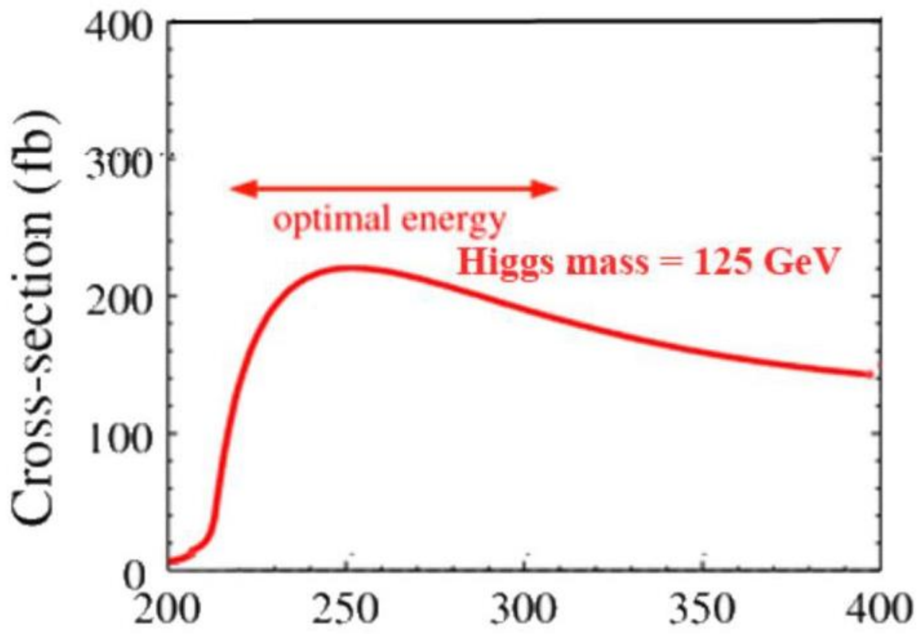
Period	Energy (GeV)	Luminosity (pb^{-1})
1995	130/136	6.2
1996	161	12.1
1996	172	11.3
1997	183	63.8
1998	189	196.4
1999	192	30.

$m_H^{max} = 98\ GeV$

e^+e^- physics potential



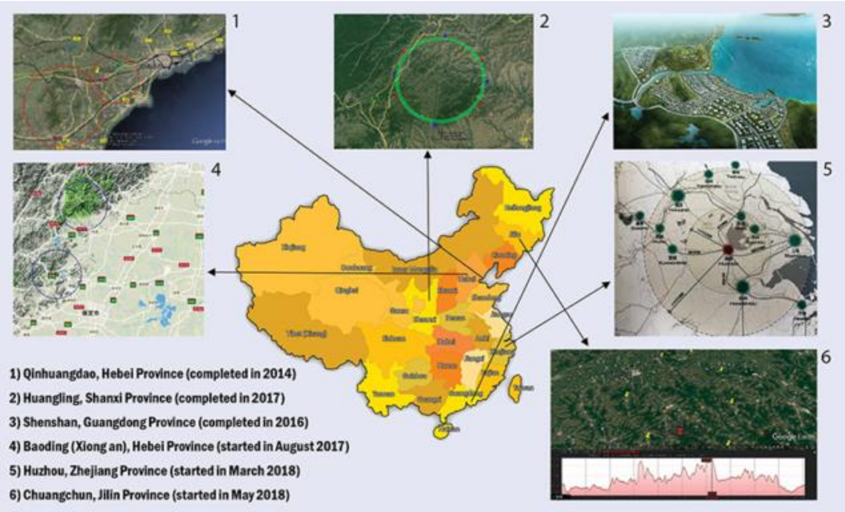


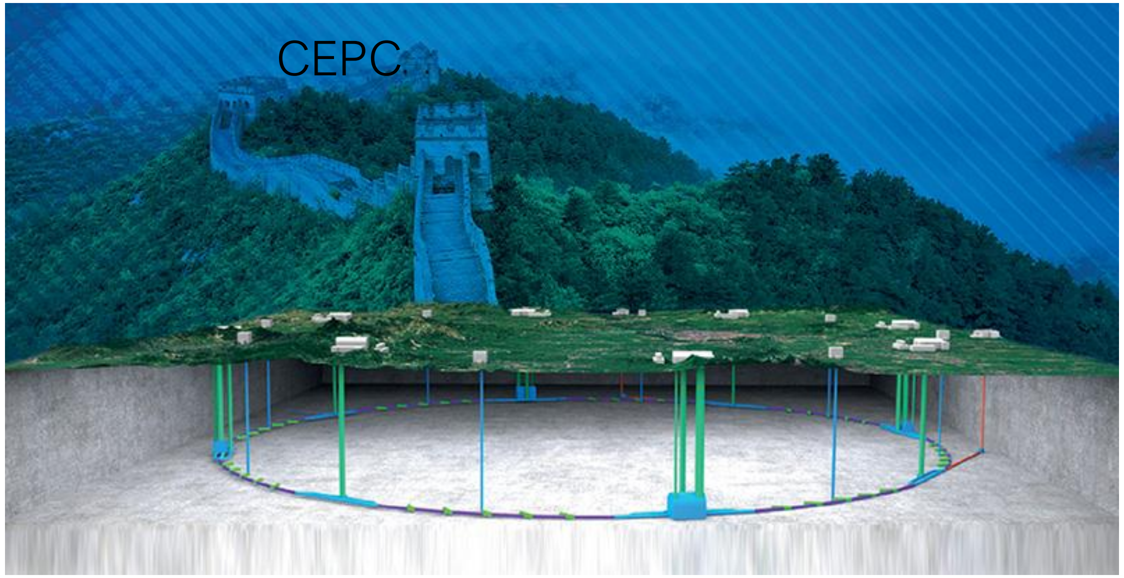


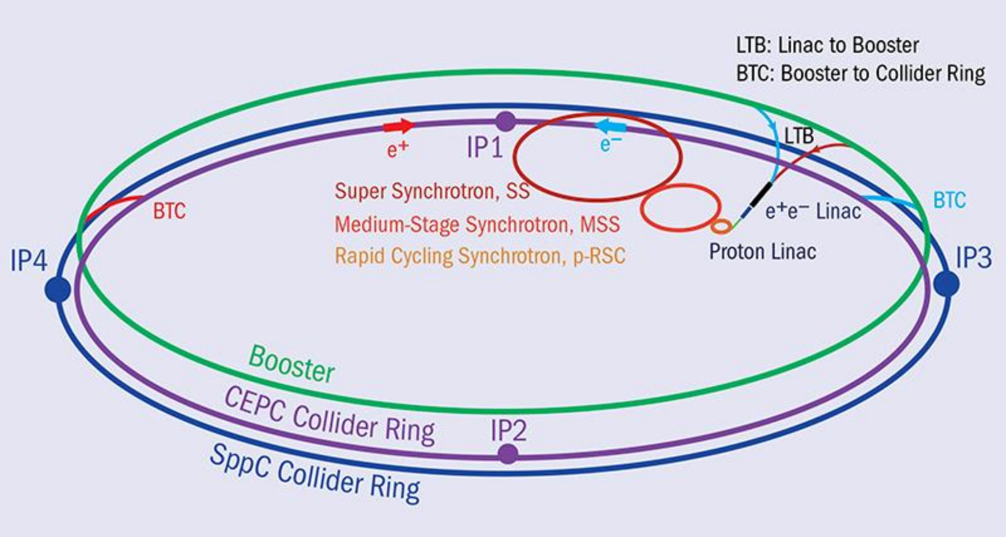
ZH reaction would give a Higgs factory

Need: $E_{cm} > m_Z + m_H = 90 + 125$ GeV $=215$ GeV

One Option: CEPC







SppC pp machine: construction in around 2040 and be completed by the mid-2040s. 75 TEV CMS energy!!

CECP: Higgs Factory

- → one million clean Higgs bosons in10 years.
- The couplings between the Higgs boson and other particles accuracy of 0.1–1% , ~10 times better than that expected of the high-luminosity LHC.
- By lowering the CMS energy to that of the Z pole at around 90 GeV, could produce at least 10 billion Z bosons per year.
- As a super Z – and W – factory, rare decays and precision of electroweak measurements.